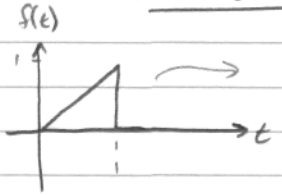
HW #3 Solutions

1)



$x: 0 \to 1 \quad f(t) = t$

$= 0$ otherwise.

$$F(\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} f(t)\, dt$$

$$= \int_0^1 t\, e^{-j\omega t}\, dt \qquad \text{TABLE}$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \left(t + \frac{1}{j\omega} \right) \right]_0^1$$

$$= \frac{e^{-j\omega}}{-j\omega} \left(1 - \frac{1}{j\omega} \right) - \left(\frac{1}{-j\omega} \left(\frac{1}{j\omega} \right) \right)$$

$$= e^{-j\omega} \left(\frac{1}{-j\omega} - \frac{1}{\omega^2} \right) - \frac{1}{\omega^2}$$

2. a

$$F(\omega) = \frac{10}{j\omega(j\omega+2)} = \frac{A}{j\omega} + \frac{B}{j\omega+2}$$

PF: $Aj\omega + 2A + Bj\omega = 10$

Re: $2A = 10 \Rightarrow A = 5$

Im: $A + B = 0 \quad B = -5$

$$F(\omega) = \frac{5}{j\omega} - \frac{5}{j\omega+2} \Longrightarrow f(t) = 2.5\,\mathrm{sgn}(t) + 5e^{-2t}u(t)$$

b. $$F(\omega) = \frac{4-j\omega}{\omega^2 - 3j\omega - 2} = \frac{4-j\omega}{(-2-j\omega)(1+j\omega)} = \frac{j\omega-4}{(j\omega+2)(j\omega+1)}$$

$$= \frac{A}{j\omega+2} + \frac{B}{j\omega+1} \Longrightarrow Aj\omega + A + Bj\omega + 2B = j\omega - 4$$

Re: $A + 2B = -4$

Im: $A + B = 1 \quad A = 1 - B$

$(1-B) + 2B = -4$

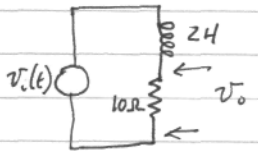
$1 + B = -4 \Rightarrow B = -5$

$A = 6$

$$= \frac{6}{j\omega+2} - \frac{5}{j\omega+1}$$

$$f(t) = \left(6e^{-2t} - 5e^{-t}\right)u(t)$$

3.



$$H(\omega) = \frac{10}{10 + 2j\omega} = \frac{5}{5 + j\omega}$$

Should convert to this form for table!

$$v_i(t) = u(t) \Rightarrow V_i(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\therefore V_o(\omega) = V_i(\omega) H(\omega) = \underbrace{\frac{5\pi\delta(\omega)}{5 + j\omega}} + \underbrace{\frac{5}{(5+j\omega)j\omega}}$$

$$\mathcal{F}^{-1}\left(\frac{5\pi\delta(\omega)}{5+j\omega}\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{5\pi}{(5+j\omega)}\delta(\omega)e^{+j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\left[\frac{5\pi}{5+0}\right] = \frac{1}{2}$$

$$\frac{A}{(5+j\omega)} + \frac{B}{j\omega} \Rightarrow \quad 5B = 5 \Rightarrow B = 1, \quad A + B = 0 \Rightarrow A = -1$$

$$-\frac{1}{5+j\omega} + \frac{1}{j\omega} \xrightarrow{\mathcal{F}^{-1}} -e^{-5t}u(t) + \frac{1}{2}\,\mathrm{sgn}(t)$$

$$\therefore \quad v_o(t) = -e^{-5t}u(t) + \underbrace{\frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t)}_{= u(t)}$$

$$= \underline{\left(1 - e^{-5t}\right)u(t)}$$

4. $H(\omega) = \frac{5}{5+j\omega}$ as before

$$v_i(t) = e^{-10t}u(t) \Longrightarrow V_i(\omega) = \frac{1}{10+j\omega}$$

$$V_o(\omega) = \frac{5}{(5+j\omega)(10+j\omega)} = \frac{A}{(5+j\omega)} + \frac{B}{(10+j\omega)}$$

Re: $10A + 5B = 5 \Longrightarrow 2A + B = 1$

Im: $A + B = 0 \Rightarrow A = -B$ ↘

$A = 1, \; B = -1$

$$V_o(\omega) = \frac{1}{(5+j\omega)} - \frac{1}{(10+j\omega)}$$

$$\therefore \; v_o(t) = \left(e^{-5t} - e^{-10t}\right)u(t)$$