HW # 4  Solns

1. \[ x(t) = \cos(\frac{2\pi \times 10^2 t}{1 \text{ MHz}}) + \sin(\frac{2\pi \times 10^2 t}{10 \text{ MHz}}) + N(t) \]

a) Technical minimum is
20 MHz, but \( \sim 100 \text{ MHz} \) might be more appropriate.

b) Maximum is limited by anti-aliasing - no point sampling higher than cutoff given by RC filter.

c) Let's choose \( f_{\text{sam}} = 100 \text{ MHz} \)

We want \( A(100 \text{ MHz}) = 0.5 \)

\[ A(f) = \frac{1}{\sqrt{1 + (\frac{f}{f_0})^2}} \]

When \( f_0 = \frac{1}{2\pi RC} \)

\[ 0.5 = \frac{1}{\sqrt{1 + (\frac{100}{f_0})^2}} \implies 1 + (\frac{100}{f_0})^2 = (\frac{1}{0.5})^2 \]

\[ (\frac{f_0}{100})^2 = (\frac{1}{0.5})^2 - 1 \]

\[ f_0 = \left( \frac{100}{(\frac{1}{0.5})^2 - 1} \right)^{1/2} = 5.74 \text{ MHz} \]
\[ C = \frac{1}{2 \pi R f_0} = \frac{1}{(2\pi)(100 \times 57.7 \times 10^6)} \]
\[ = 2.7 \times 10^{-9} F \]
\[ = 2.7 \text{ pF} \]

Quick check: what is \( A(10 \text{ MHz}) \)?

\[ A = \frac{1}{\sqrt{1 + (\frac{10}{57.7})^2}} = 0.985 \Rightarrow \text{largely unaffected} \]
2. Laser scanner @ \( 16 \text{Hz/min} = \frac{16.67 \text{ MHz}}{3} \)

300 MHz range \( \Rightarrow \) it takes 18 seconds to span range.

Clearly 8 MHz peak is traversed the fastest

\[ t_{8 \text{MHz}} = \frac{8 \text{ MHz}}{16.67 \text{ MHz/s}} = 0.485 \text{ s} \approx 0.5 \text{ s} \]

a) To get good detail sampling rate should be

\[ f_{\text{samp}} \gg \frac{1}{0.5} \]

\[ f_{\text{samp}} = \frac{10}{0.5} \approx 20 \text{ Hz} \]

b) \( f_0 = 20 \text{ Hz} \)

\[ f = \frac{1}{2\pi f_0} \]

If \( C = 0.1 \mu F \)

\[ R = \frac{1}{2\pi \times (20) \times (0.1 \times 10^{-6})} \approx 80 \text{ k\Omega} \]