LAPLACE TRANSFORM HW

1. 15.28

a) $$F(s) = \frac{2(s+1)}{(s+3)(s+5)} = \frac{A}{(s+3)} + \frac{B}{(s+5)}$$

$$\rightarrow \quad s(B+A) + 3B + 5A = 2s + 2$$

$$A + B = 2 \Rightarrow B = 2 - A$$

$$\frac{5}{3}A + B = \frac{2}{3}$$

$$\frac{5}{3}A + 2 - A = \frac{2}{3}$$

$$\frac{2}{3}A = \frac{2}{3} - 2 \Rightarrow A = -2$$

$$\therefore B = 4$$

$$F(s) = \frac{-2}{(s+3)} + \frac{4}{(s+5)} \quad \mathcal{L}^{-1}$$

$$F(t) = -2e^{-3t} + 4e^{-5t}$$

b) $$H(s) = \frac{6s+22}{(s+1)(2s^2+4s+10)} = \frac{3s+11}{(s+1)\underbrace{(s^2+2s+5)}_{\text{complete the square}}}$$

$$(s+a)^2 + b^2 = s^2 + 2s + 5$$

$$s^2 + 2sa + a^2 + b^2 = s^2 + 2s + 5$$

$$\therefore \quad a = 1 \quad , \quad b = 2$$

$$H(s) = \frac{3s+11}{(s+1)\left[(s+1)^2 + 2^2\right]}$$

$$H(s) = \frac{A}{(s+1)} + \frac{Bs+C}{\left[(s+1)^2+2^2\right]}$$

remember: $= s^2 + 2s + 5$

$$A\overbrace{\left(s^2+2s+5\right)} + (Bs+C)(s+1) = 3s+11$$

$$As^2 + 2As + 5A + Bs^2 + Bs + Cs + C = 3s+11$$

$$\begin{aligned} s^2:&\quad A + B = 0 \\ s:&\quad 2A + B + C = 3 \\ \varnothing:&\quad 5A + C = 11 \end{aligned} \quad = \quad \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 11 \end{bmatrix}}$$

$$\hookrightarrow \quad A = 2, \quad B = -2, \quad C = 1$$

$$H(s) = \frac{2}{(s+1)} - \frac{(2s-1)}{\left[(s+1)^2+2^2\right]}$$

$$\begin{aligned} (2s-1) &= 2(s+1) - b \\ 2s-1 &= 2s+2-b \\ -3 &= -b \\ b &= 3 \end{aligned}$$

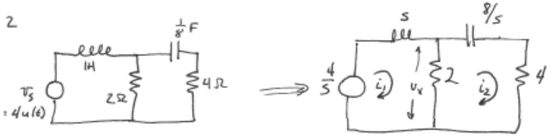
$$= \frac{2}{(s+1)} - \frac{2(s+1)-3}{\left[(s+1)^2+2^2\right]}$$

$$= \frac{2}{(s+1)} - \frac{2(s+1)}{(s+1)^2+2^2} + \frac{3\cdot 2}{\left[(s+1)^2+2^2\right]2}$$

$$\therefore \quad H(t) = 2e^{-t} - 2e^{-t}\cos 2t + \tfrac{3}{2}e^{-t}\sin 2t$$

$$= 2e^{-t}\left(1 - \cos 2t + \tfrac{3}{4}\sin 2t\right)$$

2. 16.2



loop 1: $\frac{4}{s} = i_1(s+2) - 2i_2$

loop 2: $0 = i_2(6+\frac{8}{s}) - 2i_1 \Rightarrow i_1 = i_2(3 + 4/s)$

$v_x = (i_1 - i_2)\,2$

$$\frac{4}{s} = i_2\left(3+\frac{4}{s}\right)(s+2) - 2i_2$$

$$\frac{4}{s} = i_2\left(8 + 3s + \frac{8}{s}\right) \Rightarrow 4 = i_2(3s^2+8s+8)$$

$$i_2 = \frac{4}{(3s^2+8s+8)}$$

$$i_1 = \frac{4}{(3s^2+8s+8)} \cdot \left(3+\frac{4}{s}\right)$$

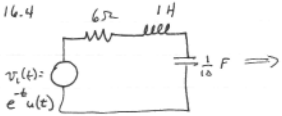
$$V_x(s) = 2\left[3+\frac{4}{s}-1\right]\frac{4}{(3s^2+8s+8)} = \frac{16\left(1+\frac{2}{s}\right)}{(3s^2+8s+8)}$$

$$v_x(s) = \frac{16(2+s)}{s(3s^2+8s+8)} = \frac{\frac{16}{3}(s+2)}{s\left(s^2+\frac{8}{3}s+\frac{8}{3}\right)}$$

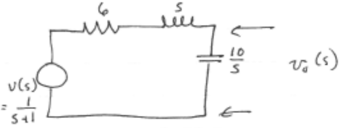
PARTIAL FRACTIONS + COMP. THE SQUARE...

$$v_x(t) = 4\left(1 - e^{-\frac{4t}{3}}\cos\frac{\sqrt{8}}{3}t\right)$$

3. 16.4



$\Longrightarrow$



$$H(s) = \frac{(10/s)}{\left(\frac{10}{s} + s + 6\right)} = \frac{10}{s^2 + 6s + 10}$$

$$V_o(s) = \frac{10}{(s+1)(s^2+6s+10)} = \frac{A}{(s+1)} + \frac{Bs+C}{(s^2+6s+10)}$$

$$(A+B)s^2 + s(6A+B+C) + A10 + C = 10$$

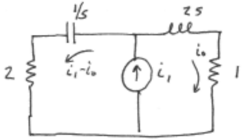
Algebra yields $\longrightarrow$ $A = 2$, $B = -2$, $C = -10$

$$V_o(s) = \frac{2}{(s+1)} - \frac{(2s+10)}{[(s+a)^2+b^2]} \Longrightarrow a = 3,\ b = 1$$

$$V_o(s) = \frac{2}{(s+1)} - \frac{2(s+5)}{(s+3)^2+1^2} = \frac{2}{s+1} - \frac{2(s+3)}{(s+3)^2+1^2} - \frac{4}{(s+3)^2+1^2}$$

$$\therefore\ V_o(t) = 2e^{-t} - 2e^{-3t}\cos \cancel{3}t - 4e^{-3t}\sin t$$

4. 16.13



$i_1(t) = e^{-2t}u(t)$

$\hookrightarrow i_1(s) = \frac{1}{s+2}$

$$H(s) = \frac{i_o(s)}{i_1(s)}$$

$$(2 + \tfrac{1}{s})(i_1 - i_o) = i_o(2s+1)$$
$$s\cdot(2 + \tfrac{1}{s})i_1 = i_o(2s + 3 + \tfrac{1}{s})\cdot s$$
$$(2s+1)i_1 = i_o(2s^2+3s+1)$$

$$\therefore H(s) = \frac{2s+1}{(2s^2+3s+1)} = \frac{\cancel{(2s+1)}}{\cancel{(2s+1)}(s+1)} = \frac{1}{s+1}$$

$$i_o(s) = H(s)i_1(s) = \frac{1}{(s+1)}\frac{1}{(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$A(s+2) + B(s+1) = 1$$
$$s(A+B) + 2A + B = 1 \implies A = -B, \ \boxed{A = 1, \ B = -1}$$

$$i_o(s) = \frac{1}{(s+1)} - \frac{1}{(s+2)}$$

$$i_o(t) = \left[e^{-t} - e^{-2t}\right]u(t)$$