Author:_____ **Division Exploration**

Group Members:

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1. Max says that $\frac{\frac{2}{3}}{\frac{3}{4}}$ is not a rational number because it is not an integer over a non-zero integer. Molly says it is a rational number. Who is right and why?

   $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$

   A rational number is any number that can be written as an integer over an integer. Since, $\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{8}{9}$, it is a rational number so Molly is right.

2. Let a, b and c be integers. Is it true that $\frac{a}{\frac{b}{c}} = \frac{\frac{a}{b}}{c}$. If it is true, prove it. If not, give a counterexample.

   No, this is not always true. Here is an example.

   $\frac{2}{\frac{3}{4}} = 2 \cdot \frac{4}{3} = \frac{8}{3}$

   $\frac{\frac{2}{3}}{4} = \frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

3. If you rewrote the above question in terms of division what property of division am I really asking about?

   $\frac{a}{\frac{b}{c}} = a \div \frac{b}{c} = a \div (b \div c)$

   $\frac{\frac{a}{b}}{c} = \frac{a}{b} \div c = (a \div b) \div c$

   So this is just asking whether the Associative property of division holds, and of course it doesn't.

4. (a) Puppies at the SPCA need $1\frac{3}{4}$ teaspons of vitamins every day. Sally noticed that there are only $7\frac{2}{3}$ teaspoons of vitamins left in the jar. How many puppies can Sally give vitamins to?

   $7\frac{2}{3} \div 1\frac{3}{4} = \frac{23}{3} \div \frac{7}{4} = \frac{23}{3} \cdot \frac{4}{7} = \frac{92}{21} = 4\frac{8}{21}$

   So Sally can feed 4 puppies.

   (b) The answer to $7\frac{2}{3} \div 1\frac{3}{4}$ is $4\frac{8}{21}$. What does $\frac{8}{21}$ measure in the previous problem?

   There is $\frac{8}{21}$ of a vitamin serving left over.

   (c) How can you use $\frac{8}{21}$ to figure out how many teaspoons of vitamins are left after Sally feeds 4 puppies?

   We have $\frac{8}{21}$ of a serving and a serving is $1\frac{3}{4}$ teaspoons. So we need to find $\frac{8}{21}$ of $1\frac{3}{4}$.

   $\frac{8}{21} \cdot 1\frac{3}{4} = \frac{8}{21} \cdot \frac{7}{4} = \frac{2}{3}$

   There will be $\frac{2}{3}$ of a teaspoon left.

5. (a) You give Max the problem $\frac{2}{3} \div \frac{7}{5}$. He computes it in the following way. Is Max's answer correct?

$$\frac{2}{3} \div \frac{7}{5} = \frac{2 \div 7}{3 \div 5} = \frac{\frac{2}{7}}{\frac{3}{5}} = \frac{2}{7} \cdot \frac{5}{3} = \frac{10}{21}$$

(b) Compute the problem $\frac{4}{5} \div \frac{8}{3}$ using Max's method.

$$\frac{4}{5} \div \frac{8}{3} = \frac{4 \div 8}{5 \div 3} = \frac{\frac{4}{8}}{\frac{5}{3}} = \frac{4}{8} \cdot \frac{3}{5} = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$$

(c) Will Max always get the right answer doing it this way? If so, prove that $\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$. If not, give Max an example to show that his method will not always work.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

$$\frac{a \div c}{b \div d} = \frac{\frac{a}{c}}{\frac{b}{d}} = \frac{a}{c} \div \frac{b}{d} = \frac{a}{c} \cdot \frac{d}{b} = \frac{ad}{cb} = \frac{ad}{bc}$$

Thus $\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$.

6. (a) You ask Max to compute $\frac{2}{3} \div \frac{1}{9}$. He does the following. Does Max get the correct answer?

$$\frac{2}{3} = \frac{6}{9}$$

Since $6 \div 1 = 6$, then $\frac{2}{3} \div \frac{1}{9} = 6$

(b) You then ask him to compute $\frac{2}{3} \div \frac{1}{4}$. He uses the same method, but now he needs to change both fractions. Does Max get the correct answer?

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{1}{4} = \frac{3}{12}$$

Since $8 \div 3 = \frac{8}{3} = 2\frac{2}{3}$, then $\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3}$.

(c) Use Max's method to compute $\frac{5}{6} \div \frac{2}{5}$.

$$\frac{5}{6} = \frac{25}{30}$$

$$\frac{2}{5} = \frac{12}{30}$$

$$\frac{5}{6} \div \frac{2}{5} = 25 \div 12 = \frac{25}{12} = 2\frac{1}{12}$$

(d) The method Max uses consists of two steps. First get a common denominator, then divide the numerators. Prove that if you divide two fractions with a common denominator, you need only divide the numerators. In other words, prove that $\frac{a}{c} \div \frac{b}{c} = a \div b$.

$$\frac{a}{c} \div \frac{b}{c} = \frac{a}{c} \cdot \frac{c}{b} = \frac{ac}{bc} = \frac{a}{b} = a \div b$$

(e) Explain to Max why his method works.

There are lots of different ways to explain this, but here's one:

To divide two fractions you must invert and multiply. If the two fractions have a common denominator, then after inverting the "common denominators" will cancel each other out. This leaves you with just dividing the numerators.