1. **True or False.** Determine whether the following statements are true or false. If the statement is always true, give a brief justification. If the statement is sometimes false, give a counterexample or brief justification.

   T **F** If $\frac{a}{b}$ is a rational number, then $\left(\frac{a}{b}\right)^4 = \frac{a^4}{b^4}$.

   $$\left(\frac{a}{b}\right)^4 = \frac{a}{b}\cdot\frac{a}{b}\cdot\frac{a}{b}\cdot\frac{a}{b} = \frac{a\cdot a\cdot a\cdot a}{b\cdot b\cdot b\cdot b} = \frac{a^4}{b^4}$$

   T **F** Max computes $2\frac{1}{4}\cdot 3\frac{3}{5}$ in the following way. Max's method and answer are correct.

   $2\cdot 3 = 6$ and $\frac{1}{4}\cdot\frac{2}{5} = \frac{3}{20}$

   So $2\frac{1}{4}\cdot 3\frac{3}{5} = 6\frac{3}{20}$

   $$2\frac{1}{4}\cdot 3\frac{3}{5} = \frac{9}{4}\cdot\frac{18}{5} = \frac{162}{20} = \frac{81}{10} = 8\frac{1}{10}$$

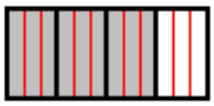
   Since the correct answer is $8\frac{1}{10}$ Max's answer and method are incorrect.

   T **F** -2 is a rational number. Since $-2 = \frac{-2}{1}$, -2 is a rational number.

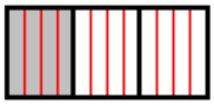
   T **F** The Fundamental Law of Fractions can be used to show $\frac{-a}{b}$ equals $\frac{a}{-b}$.

   $$\frac{-a}{b} = \frac{-a\cdot -1}{b\cdot -1} = \frac{a}{-b}$$

2. (a) Use a parts of the whole model to find the sum $\frac{3}{4}+\frac{1}{3}$.

   I represented $\frac{3}{4}$ using parts of the whole, then broke each $\frac{1}{4}$ into three equal pieces.

   I represented $\frac{1}{3}$ using parts of the whole, then broke each $\frac{1}{3}$ into four equal pieces.

   Now all of the pieces are the same size, and there are 12 pieces in a whole.
   There is a total of 13 shaded pieces.
   So $\frac{3}{4}+\frac{1}{3} = \frac{13}{12}$.

   (b) Find the sum $\frac{3}{4}+\frac{1}{3}$ algebraically. Be sure to justify each equality.

   $$\frac{3}{4}+\frac{1}{3} \overset{(i)}{=} \frac{3\cdot 3}{4\cdot 3}+\frac{1\cdot 4}{3\cdot 4} \overset{(ii)}{=} \frac{9}{12}+\frac{3}{12} \overset{(iii)}{=} \frac{13}{12}$$

   Justifications:

(i) Fundamental Law
(ii) Multiplication in $\mathbb{Z}$
(iii) Addition in $\mathbb{Q}$

(c) Choose **one** of the steps in the algebraic method and explain where it showed up in your parts of the whole model.

The first step shows up when we break each of the pieces down more (shown in red above).

The second step shows up when counting how many new shaded pieces in each model.

The third step shows up when totaling all the shaded pieces.

3. For each of the following word problems below, circle which operation would give the solution. You do not need to solve the problem.

(a) Students in Mr. Jones' class who wish to take a foreign language are allowed to take only one of the following: Spanish, French or German. One half of Mr. Jones' class is taking Spanish and two fifths of the class is taking French. What fraction of the class is taking Spanish or French?

$\boxed{\frac{1}{2}+\frac{2}{5}}$ $\frac{1}{2}-\frac{2}{5}$ $\frac{1}{2}\cdot\frac{2}{5}$ Not Listed

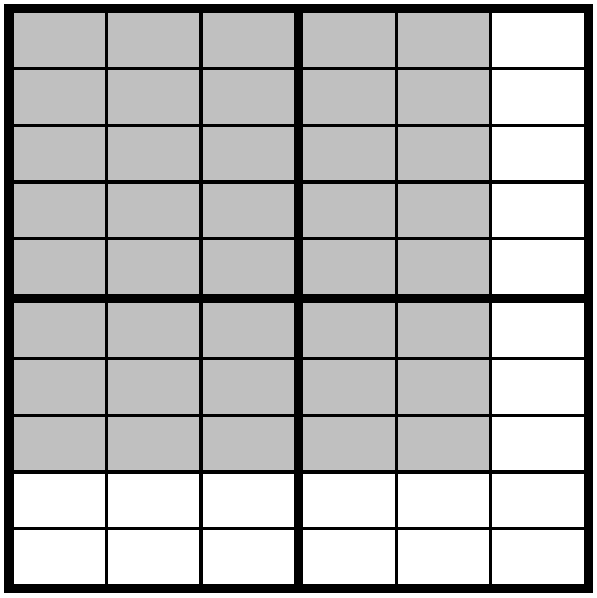
(b) Students in Mr. Jones' class who wish to take a foreign language are allowed to take only one of the following: Spanish, French or German. One half of Mr. Jones' class takes a foreign language. Two fifths of his class is taking Spanish. What fraction of the class is taking a foreign language other than Spanish?

$\frac{1}{2}+\frac{2}{5}$ $\boxed{\frac{1}{2}-\frac{2}{5}}$ $\frac{1}{2}\cdot\frac{2}{5}$ Not Listed

(c) Students in Mr. Jones' class who wish to take a foreign language are allowed to take only one of the following: Spanish, French or German. One half of Mr. Jones' class takes a foreign language. Two fifths of the students taking a foreign language are taking Spanish. What fraction of Mr. Jones' class is taking Spanish?

$\frac{1}{2}+\frac{2}{5}$ $\frac{1}{2}-\frac{2}{5}$ $\boxed{\frac{1}{2}\cdot\frac{2}{5}}$ Not Listed

4. Below is an area model for a certain multiplication problem.



   (a) Without figuring out what the multiplication problem is, what is the **answer** to the multiplication problem? Be sure to give enough explanation so that I can see you are getting the answer from the picture.

   There are 40 shaded pieces, which tells us the numerator. There are 15 pieces in a whole, which tells us the denominator. Therefore the answer to the multiplication is $\frac{40}{15}$.

   (b) What is the multiplication problem?

   The rectangle has dimensions $\frac{8}{5}$ by $\frac{5}{3}$. Therefore the multiplication problem is $\frac{8}{5} \cdot \frac{5}{3}$.

5. State and solve a word problem whose solution is $2\frac{2}{3} \cdot 3\frac{1}{2}$.

6. Without relying on the accuracy of a drawing, getting a common denominator or using the cross multiply algorithm, determine which is larger $\frac{9}{10}$ or $\frac{10}{11}$. Explain your reasoning.

   In the parts of the whole model for each of the fractions there is one piece needed to complete the whole. The pieces for $\frac{9}{10}$ are bigger than the pieces in $\frac{10}{11}$. Therefore the model for $\frac{9}{10}$ has a bigger piece left over, therefore $\frac{10}{11}$ takes up more space and hence is the bigger fraction.

7. (a) Find 3 rational numbers between $\frac{a}{5}$ and $\frac{b}{5}$. (You may assume a is smaller than b.)

$$\frac{a}{5} = \frac{a \cdot 4}{5 \cdot 4} = \frac{4a}{20}$$
$$\frac{b}{5} = \frac{b \cdot 4}{5 \cdot 4} = \frac{5a}{20}$$

   Therefore three numbers between are $\frac{4a+1}{20}, \frac{4a+2}{20}, \frac{4a+3}{20}$.

(b) Look at the last number you listed in your answer above. Explain how you know that number is smaller than $\frac{b}{5}$.

Since a is less than b, we know their difference is at least 1. When we use multiplier 4, we then know that the difference between $4a$ and $4b$ is at least 4, which means at least 3 integers fit in between $4a$ and $4b$, thus $4a + 3$ (which is the third number) will be less than $4b$. Thus $\frac{4a+3}{20}$ is less than $\frac{4b}{20}$ which is equal to $\frac{b}{5}$.

8. Prove that if $\frac{a}{b}$ is a rational number greater than one, then $\frac{2}{5} \cdot \frac{a}{b} > \frac{2}{5}$.

We need to compare $\frac{2}{5} \cdot \frac{a}{b}$ and $\frac{2}{5}$, so we compute the product and get a common denominator.

$$\frac{2}{5} \cdot \frac{a}{b} = \frac{2a}{5b}$$

$$\frac{2}{5} = \frac{2b}{5b}$$

Since $\frac{a}{b} > 1$, we know $a > b$. Thus $2a > 2b$, so $\frac{2a}{5b} > \frac{2b}{5b}$. Therefore $\frac{2}{5} \cdot \frac{a}{b} > \frac{2}{5}$.