1. **True or False.** Determine whether the following statements are true or false. If the statement is always true, give a **brief** justification. If the statement is sometimes false, give a counterexample or **brief** justification.

[T] **F** $\sqrt{160}$ is an irrational number.

$\sqrt{160}$ is an irrational number because 160 is not a perfect square.

T **[F]** For all natural numbers a, b and c, $\frac{\frac{a}{b}}{c} = \frac{a}{\frac{b}{c}}$.

$$\frac{1}{\frac{2}{3}} = 1 \cdot \frac{3}{2} = \frac{3}{2}$$

$$\frac{\frac{1}{2}}{3} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

T **[F]** There exist natural numbers a and b such that $18b^2 = a^2$.

The prime factorization of $18b^2$ has an odd amount of twos however a^2 has an even amount of twos. Therefore $18b^2$ cannot equal a^2.

2. Give an example for each of the following.

   (a) A real number that is not a rational number. $\sqrt{2}$

   (b) A fraction that is not a rational number. $\frac{\sqrt{2}}{3}$

   (c) A terminating decimal greater than $.\overline{9}$. 1.1

3. Jolynn has $5\frac{1}{3}$ feet of yarn. She is doing a craft project with her preschool class and needs $\frac{1}{2}$ foot for each student.

   (a) How many students will be able to do the craft?

   $$5\frac{1}{3} \div \frac{1}{2} = \frac{16}{3} \div \frac{1}{2} = \frac{16}{3} \cdot \frac{2}{1} = \frac{32}{3} = 10\frac{2}{3}$$

   10 students will be able to do the craft.

   (b) Will she have any yarn left over? If so, how many feet?

   $$\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$$

   There will be $\frac{1}{3}$ of a foot left over.

4. Without dividing the numerator by the denominator, determine whether the following can be expressed as a terminating decimal, repeating decimal, or neither. Please give a **brief** explanation. (You **do not need** to find the actual decimal.)

   (a) $\dfrac{\sqrt{3}}{20}$

   Since $\sqrt{3}$ is irrational, so is $\dfrac{\sqrt{3}}{20}$. So $\dfrac{\sqrt{3}}{20}$ is neither a terminating decimal nor a repeating decimal.

   (b) $\dfrac{7}{35} = \dfrac{1}{5}$

   After reducing, this rational number has only a five in the denominator, so it can be expressed as a terminating decimal.

   (c) $\dfrac{4}{21}$

   This rational number has a 3 in the prime factorization of the denominator. Since the denominator has something other than twos and/or fives in the prime factorization, $\frac{4}{21}$ will be a repeating decimal.

   (d) $\dfrac{7}{30}$

   This rational number has a 3 in the prime factorization of the denominator. Since the denominator has something other than twos and/or fives in the prime factorization, $\frac{7}{30}$ will be a repeating decimal.

5. If possible, write the following decimals as a rational fraction in simplest form. If it is not possible, state why. Be sure to show all steps.

   (a) $.036 = 36 \cdot 10^{-3} = \dfrac{3}{6} \cdot \dfrac{1}{10^3} = \dfrac{36}{1000} = \boxed{\dfrac{9}{250}}$

   (b) $.217171717\ldots$

$$\begin{aligned} 1000(.2\overline{17}) &= 217.17171717\ldots \\ 10(.2\overline{17}) &= 2.17171717\ldots \\ 1000(.2\overline{17}) - 10(.2\overline{17}) &= 217.17171717\ldots - 2.17171717\ldots \\ 990(.2\overline{17}) &= 215 \\ .2\overline{17} &= \frac{215}{990} = \boxed{\frac{43}{198}} \end{aligned}$$

   (c) $.329229222922229\ldots$

   This is a non-terminating, non-repeating decimal and hence is irrational. Therefore it cannot be written as a rational fraction.

6. You take 80% of a number, then take 70% of the result. Will you get the same answer if you first take 70% of the number, then 80% of the result? If the answer is yes, prove it. If it is no, give a counterexample.

   80% of 70% of x equals $.80(.70)x$

   70% of 80% of x equals $.70(.80)x$.

   Therefore these two things are equal due to the commutative property of multiplication.

7. (a) Without dividing, write $\frac{3}{80}$ as a decimal. Be sure to show all work.

$$80 = 2^4 \cdot 5$$

$$\frac{3}{80} = \frac{3 \cdot 5^3}{80 \cdot 5^3} = \frac{375}{10^4} = 375 \cdot 10^{-4} = \boxed{.0375}$$

(b) In the above process, you used the Fundamental Law. Explain how you are able to decide which multiplier to use without guess and check.

The prime factorization of 80 has four 2s and one 5. We need to make the 2s and 5s match up. Therefore we needed three more 5s. So we used 5^3 for the multiplier.

8. Consider the problem $2.51 \div 3.2$.

(a) You have just finished teaching your students how to divide a decimal by a whole number. Now explain to your students how to compute the above quotient. (Please do not re-explain dividing decimals by whole numbers.)

i. Move the decimal point one place to the right in the divisor.
ii. Move the decimal point one place to the right in the dividend.
iii. Now the problem is a decimal divided by a whole number, which you know how to do.

(b) Explain why the algorithm you explained above is correct.

You told the students to change the problem $2.51 \div 3.2$ into the problem $25.1 \div 32$. So we need to justify why the answer to these two problems are equal.

$$2.51 \div 3.2 = \frac{2.51}{3.2} = \frac{2.51 \cdot 10}{3.2 \cdot 10} = \frac{25.1}{32} = 25.1 \div 32$$

Therefore these two problems are equivalent due to the Fundamental Law of Fractions.

9. Prove that $\frac{\sqrt{8}}{2}$ is irrational.

Below are two different solutions:

(a) (First way) Suppose $\frac{\sqrt{8}}{2}$ is rational. Then there are integers a and b so that $\frac{\sqrt{8}}{2} = \frac{a}{b}$. Therefore we would have the following.

$$\frac{\sqrt{8}}{2} = \frac{a}{b}$$

$$\sqrt{8} = \frac{2a}{b}$$

Now, $\sqrt{8}$ is irrational, but $\frac{2a}{b}$ is rational. Since a rational number cannot equal an irrational number, the above equation must not be true, and since it came from the equation $\sqrt{8} - 2 = \frac{a}{b}$ must not be true either. Therefore $\sqrt{8} - 2$ cannot be expressed in rational form, so $\sqrt{8} - \frac{a}{b}$ is irrational.

(b) (Second way)

$$\frac{\sqrt{8}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

We know $\sqrt{2}$ is irrational since 2 is not a perfect square. Therefore $\frac{\sqrt{8}}{2}$ is irrational.