Author:_____ **Exponents**

Group Members:

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1. Let a be a rational number. What do the following equal? You may not use any exponent rules (other than the definition of exponents).
   (a) $a^2 \cdot a^3 = (a \cdot a)(a \cdot a \cdot a) = a \cdot a \cdot a \cdot a \cdot a = a^5$
   (b) $\dfrac{a^5}{a^2} = \dfrac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = \dfrac{a \cdot a \cdot a}{1} = a \cdot a \cdot a = a^3$
   (c) $(a^2)^3 = (a \cdot a)^3 = (a \cdot a)(a \cdot a)(a \cdot a) = a \cdot a \cdot a \cdot a \cdot a \cdot a = a^6$

2. Complete the following exponent rules.

   $a^n a^m = a^{n+m}$ $\qquad$ $\dfrac{a^n}{a^m} = a^{n-m}$ $\qquad$ $(a^n)^m = a^{nm}$

3. Let $\frac{a}{b}$ be a rational number. Is it true that $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$. If yes, explain why. If no, give a counterexample.

   Yes this is true.

   $$\left(\frac{a}{b}\right)^n = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdots \frac{a}{b}}_{n \text{ times}} = \frac{\overbrace{a \cdot a \cdots a}^{n \text{ times}}}{\underbrace{b \cdot b \cdots b}_{n \text{ times}}} = \frac{a^n}{b^n}$$

4. Simplify the following expressions into the form a^n (where n is a whole number) using only the exponent rules stated in problem #2. I have done the first one for you.
   (a) $6^3 \cdot 6^0 = 6^{3+0} = 6^3$
   (b) $6 \cdot 6^0 = 6^1 \cdot 6^0 = 6^{1+0} = 6^1 = 6$
   (c) $a \cdot a^0 = a^1 \cdot a^0 = a^{1+0} = a^1 = a$
   (d) $3^4 \cdot 3^{-1} = 3^{4+-1} = 3^3$
   (e) $3 \cdot 3^{-1} = 3^1 \cdot 3^{-1} = 3^{1+-1} = 3^0$
   (f) $\dfrac{2}{3} \cdot \left(\dfrac{2}{3}\right)^{-1} = \left(\dfrac{2}{3}\right)^1 \left(\dfrac{2}{3}\right)^{-1} = \left(\dfrac{2}{3}\right)^{1+-1} = \left(\dfrac{2}{3}\right)^0$
   (g) $\dfrac{a}{b} \cdot \left(\dfrac{a}{b}\right)^{-1} = \left(\dfrac{a}{b}\right)^1 \left(\dfrac{a}{b}\right)^{-1} = \left(\dfrac{a}{b}\right)^{1+-1} = \left(\dfrac{a}{b}\right)^0$
   (h) $3^4 \cdot 3^{-4} = 3^{4+-4} = 3^0$
   (i) $\left(\dfrac{2}{3}\right)^5 \cdot \left(\dfrac{2}{3}\right)^{-5} = \left(\dfrac{2}{3}\right)^{5+-5} = \left(\dfrac{2}{3}\right)^0$
   (j) $\left(\dfrac{a}{b}\right)^n \cdot \left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{a}{b}\right)^{n+-n} = \left(\dfrac{a}{b}\right)^0$