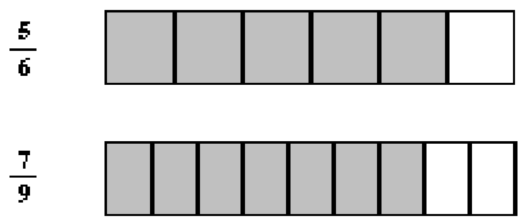
HW #1 – ch. 1 problems 1-5 – SOLUTIONS

1. Explain why the parts of the whole model cannot be used to understand the following fractions. Be sure to indicate specifically where the problem occurs in the parts of the whole model.

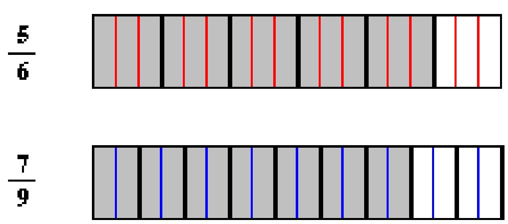
   (a) $\dfrac{-2}{3}$
   We can designate a whole and break it into 3 pieces, but we cannot shade -2 pieces.

   (b) $\dfrac{2}{-3}$
   We can designate a whole, but we cannot break it into -3 pieces.

2. (a) Use a "parts of the whole" model to determine which fraction is larger, $\dfrac{5}{6}$ or $\dfrac{7}{9}$. Be sure to explain all steps carefully.
   We begin by representing $\frac{5}{6}$ and $\frac{7}{9}$ using the same size whole.



   To compare these two fractions, I need to get the same size pieces in each model. I will break each piece in the $\frac{5}{6}$ model into 3 pieces (indicated by red below). I will break each piece in the $\frac{7}{9}$ model into 2 pieces (indicated in blue below).



   All the pieces are now the same size. $\frac{5}{6}$ is represented by 15 pieces and $\frac{7}{9}$ is represented by 14 pieces. Thus $\frac{5}{6}$ is greater than $\frac{7}{9}$.

   (b) Determine algebraically which fraction is larger, $\dfrac{5}{6}$ or $\dfrac{7}{9}$. Justify each equality.

$$\frac{5}{6} = \frac{5 \cdot 3}{6 \cdot 3} = \frac{15}{18}$$
$$\frac{7}{9} = \frac{7 \cdot 2}{8 \cdot 2} = \frac{14}{18}$$

   By looking at the equivalent fractions, since $15 > 14$ we see that $\frac{5}{6} > \frac{7}{9}$.

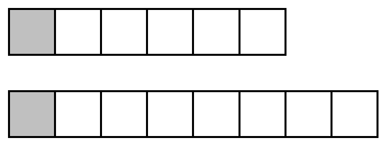
   (c) Where does the multiplier in the Fundamental Law of Fractions show up in your parts of the whole model above?
   The multiplier 3 showed up in your picture above because you broke each of the pieces in the model for $\frac{5}{6}$ into 3 equal pieces. Similarly the 2 showed up because you broke each of the pieces in the model for $\frac{7}{9}$ into 2 equal pieces.

3. Four friends want to share six candy bars fairly. Considering we do not understand what $6 \div 4$ means yet, use drawings and explain how to determine how much each friend gets.

There are lots of possibilities here. Here's one.

Break each candy bar into four pieces, which means each piece is $\frac{1}{4}$ of a candy bar. Give each friend one piece from each candy bar. This means they will have 6 pieces, but four pieces make up a whole, so they have one whole candy bar and two pieces of another. Since four pieces make up a candy bar, 2 pieces makes up half a candy bar. Thus each friend will get one and a half candy bars.

4. Max is comparing the fractions $\dfrac{1}{6}$ and $\dfrac{1}{8}$. He draws the following pictures and makes the conclusion that the two fractions are equal.



(a) According to his pictures and his conclusion, what does Max understand about fractions?

Max understands the concept of breaking a whole into equal pieces given by the denominator and that the numerator tells him how many pieces to shade. He also understands that he wants to compare the size of each of those pieces.

(b) According to his pictures and his conclusion, what does Max not understand about fractions?

He does not understand that when comparing two fractions you need to be working with the same "whole".

(c) What would you say to Max to help him.

I would tell him that a fraction does not mean anything unless you have a whole to compare it too, therefore when comparing two fractions you want to make sure you are looking at the same whole. So if he redraws his pictures where both wholes are the same size, then he will see that $\frac{1}{6}$ is bigger than $\frac{1}{8}$.

5. (a) Max claims that if we have two positive rational numbers, the one with the greatest numerator is the greatest. Is Max correct? If so, explain why. If not, explain why not and give a counterexample.

I would tell Max that he cannot only look at the numerator to determine which fraction is greater because the denominator is giving us information about how big or small the pieces are. So even though we may have a lot of pieces, if they are small it may less than a few larger pieces. Then I would give him the following example.

Consider the fractions $\dfrac{4}{7}$ and $\dfrac{3}{5}$. The first fraction has the greater numerator, however the second fraction is actually larger, which can be seen by the following.

$$\frac{4}{7} = \frac{4 \cdot 5}{7 \cdot 5} = \frac{20}{35}$$
$$\frac{3}{5} = \frac{3 \cdot 7}{5 \cdot 7} = \frac{21}{35}$$

Since $21 > 20$, then we know $\dfrac{3}{5} > \dfrac{4}{7}$.

(b) Mary claims that if we have two positive rational numbers, the one with the greatest denominator is the least. Is Mary correct? If so, explain why. If not, explain why not and give a counterexample.

I would tell Mary that she cannot only look at the denominator to determine which fraction is greater because the numerator is giving us information about how many pieces we have. So if we have a bunch of small pieces it could be more than a few larger pieces. I would give her the following example. Consider the fractions $\frac{4}{7}$ and $\frac{1}{5}$. The first fraction has the greater denominator, however the second fraction is actually smaller, which can be seen by the following.

$$\frac{4}{7} = \frac{4 \cdot 5}{7 \cdot 5} = \frac{20}{35}$$
$$\frac{1}{5} = \frac{1 \cdot 7}{5 \cdot 7} = \frac{7}{35}$$

Since $7 < 20$, then we know $\frac{1}{5} < \frac{4}{7}$.