HW #10 – ch. 3 problem #2,3 – SOLUTIONS

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2. You have just taught Max the definition of $\sqrt{\ }$, but he has several questions. (Max is not allowed to use calculators, so your explanation cannot rely on use of a calculator.)

   (a) Since $(-3)^2 = 9$, why doesn't $\sqrt{9} = -3$?

   By the definition, $\sqrt{9}$ is the nonnegative solution to $x^2 = 9$. Although -3 is a solution to the equation, -3 is not non-negative. Therefore $\sqrt{9}$ does not equal -3.

   (b) Does $\sqrt{4} + \sqrt{5} = \sqrt{4+5}$?

   There are lots of possible solutions here, but here's one:

   $$\sqrt{4} + \sqrt{5} = 2 + \sqrt{5}$$

   $$\sqrt{4+5} = \sqrt{9} = 3$$

   If $2 + \sqrt{5} = 3$, then $\sqrt{5}$ must equal 1. Since $\sqrt{5}$ does not equal 1, then $2 + \sqrt{5} \neq 3$ and hence $\sqrt{4} + \sqrt{5} \neq \sqrt{4+5}$.

   (c) Does $\sqrt{4}\sqrt{5} = \sqrt{4 \cdot 5}$?

   Note that $\sqrt{4 \cdot 5} = \sqrt{20}$ and the definition of $\sqrt{20}$ is the unique non-negative solution to $x^2 = 20$. Therefore we need only check that $\sqrt{4}\sqrt{5}$ is a solution to $x^2 = 20$. Let's check

   $$(\sqrt{4}\sqrt{5})^2 = (\sqrt{4}\sqrt{5})(\sqrt{4}\sqrt{5}) = (\sqrt{4}\sqrt{4})(\sqrt{5}\sqrt{5}) = 4 \cdot 5 = 20$$

   Since we got 20, $\sqrt{4}\sqrt{5} = \sqrt{4 \cdot 5}$

   (d) Why can't you take the square root of a negative number?

   If there is a negative number inside the square root we would need to find a number that when multiplied by itself would give a negative. This is impossible since a number multiplied by itself will always be positive (unless it's zero).

3. In light of what you explained to Max in problem 0d, Max now believes that you cannot solve the equation $\sqrt{-x} = 9$. Is Max correct? Explain.

$$\begin{aligned} \sqrt{-x} &= 9 \\ -x &= 81 \\ x &= -81 \end{aligned}$$

Therefore this equation does have a solution, namely $x = -81$. The confusion here is that Max thinks that $-x$ is a negative number, but really $-x$ means the "opposite of x", and hence is not necessarily a negative number.