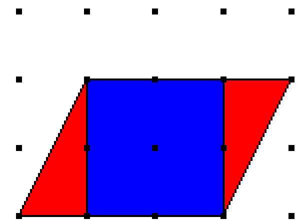
HW #11 – ch. 4 problem #1,3,5,6,7 – SOLUTIONS

1. Using the fact that the vertical and horizontal distance between consecutive dots is 1 unit, find the area of the shapes drawn on dot paper. (See attached page.) Be sure to show all work clearly (colored pencils may be useful). You may not use any area formulas.

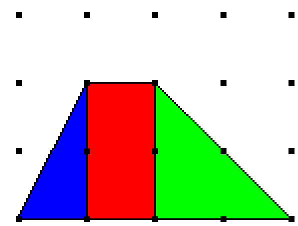
   (a) The area of the blue section is 4. Each of the red sections is half of a 1 by 2 rectangle, so each of the red sections has area 1. Thus we get

   $$A = 4 + 1 + 1 = \boxed{6}$$

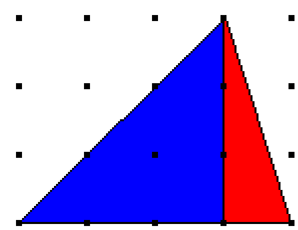
   (b) The blue section is half of a 1 by 2 rectangle, so the area of the blue section is 1. The red section has area 2. The green section is half of a 2 by 2 square, so the area of the green section is 2. Thus we get

   $$A = 1 + 2 + 2 = \boxed{5}$$

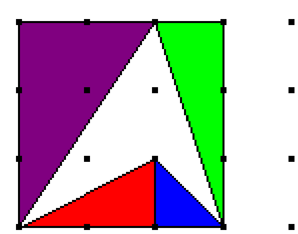
   (c) The blue section is half of a 3 by 3 square, so the area of the blue section is $\frac{1}{2} \cdot 9 = \frac{9}{2}$. The red section is half of a 1 by 3 rectangle, so the area of the red section is $\frac{1}{2} \cdot 3 = \frac{3}{2}$. Thus we get

   $$A = \frac{9}{2} + \frac{3}{2} = \boxed{6}$$

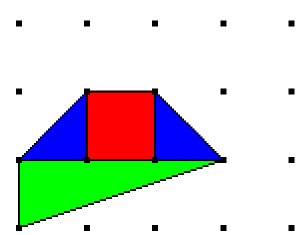
   (d) The purple section is half of a 2 by 3 rectangle, so the area of the purple section is 3. The green section is half of a 1 by 3 rectangle, so the area of the green section is $\frac{3}{2}$. The red section is half of a 1 by 2 rectangle, so the area of the red section is 1. The blue section is half of a 1 by 1 square, so the area of the blue section is $\frac{1}{2}$. To get the area of the shape, we need to subtract all the colored regions from the total 3 by 3 square on the outside. Thus we get

   $$A = 9 - 3 - \frac{3}{2} - 1 - \frac{1}{2} = \boxed{3}$$

   (e) The blue sections are each half of a 1 by 1 square, so the area of each of the blue sections is $\frac{1}{2}$. The red section is a 1 by 1 square so the area of the red section is 1. Finally, the green section is half of a 1 by 3 rectangle, so the area of the green section is $\frac{3}{2}$. Thus we get

   $$A = \frac{1}{2} + \frac{1}{2} + 1 + \frac{3}{2} = \boxed{3\frac{1}{2}}$$

2. In 1899, G. Pick discovered a very surprising theorem. We'll use the shapes above to explore his theorem.

(a) Use the table below to record your answers.

i. Let I equal the number of dots inside the shape. Find I for each of the shapes in problem 1.

ii. Let B equal the number of dots that lie on the shape. Find B for each of the shapes in problem 1.

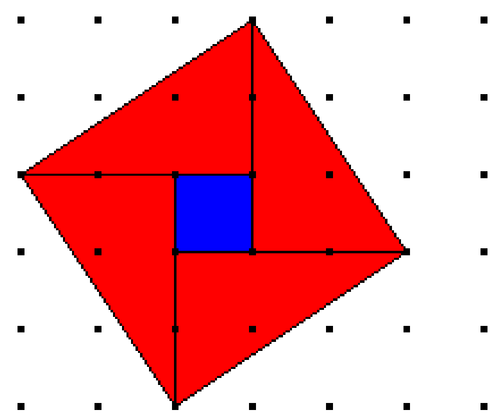
iii. Compute $I + \frac{1}{2}B - 1$ for each of the shapes in problem 1.

iv. Your answer from part (c) should match up with the areas you computed in problem 1. Do they? That is precisely what **Pick's Theorem** says: The area of a shape constructed on dot paper is equal to $I + \frac{1}{2}B - 1$.

	I	B	$I + \frac{1}{2}B - 1$
Figure 1(a)	3	8	6
Figure 1(b)	2	8	6
Figure 1(c)	3	8	6
Figure 1(d)	2	4	3
Figure 1(e)	2	5	$3\frac{1}{2}$

(b) Consider the square drawn on dot paper below.

i. Find the area of this square using the definition of area. (i.e. you may not use any area formulas) There are four triangles (shown in red) and they are each half of a rectangle containing six squares. Therefore the red portion has area $4(\frac{1}{2})(6) = 12$. Plus there is the blue square, so the area of the whole square is 13.



ii. Find the area of this square using Pick's Theorem.

$I = 12$

$B = 4$

$\text{Area} = I + \frac{1}{2}B - 1 = 12 + \frac{1}{2}(4) - 1 = 13$

3. (a) Find the dimensions of a square that has area 6.25 in^2.

Let $x =$ the length of a side in the square. So the area of the square is $x \cdot x$. Therefore we need to solve the equation $x^2 = 6.25$. The solutions to this equation are $x = \pm 2.25$, but of course we are talking about sides in a square so negative solutions don't make sense. Therefore the dimensions of a square with area 6.25 in^2 are 2.25 in by 2.25 in.

(b) Explain why it is impossible to construct a square on dot paper with area 6.25 square units.

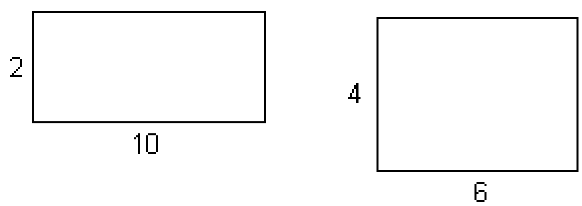
There are lots of possible explanations. Here's one:

Notice that if a square is not tilted, then you can just count up the number of squares inside the shape. Therefore the area of a non-tilted square will be a whole number.

If the square is tilted (see problem number 2) then the square can be broken down into four non-tilted right triangles and some 1 by 1 squares. The area of any non-tilted right triangle is either a whole number or half of a whole number. So the only decimals we can get would be .5's. In addition, the 1 by 1 squares all have whole number area. Therefore combining the triangles and squares, the only possible area would be a whole number or a decimal with .5. In other words, we will never get a .25 in the decimal.

5. Max is comparing two rectangles and says that the rectangle with the larger perimeter must have larger area. Will Max always be correct? If so, explain why. If not, give Max an example to see that his conclusion may be false.

Just because the perimeter is bigger doesn't necessarily mean the area will be bigger. Here's an example where the rectangle with the bigger perimeter has the smaller area.



6. For each of the questions below an example is not enough, you must show that your answer holds for *any* rectangle.

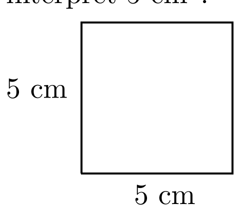
(a) If you triple the lengths of all the sides in a rectangle, what is the change in the perimeter?

Let the rectangle have dimensions x by y, then the perimeter is $P = 2x + 2y$. If you now triple the sides, the perimeter is $P = 2(3x) + 2(3y) = 3(2x + 2y)$, so we see that the perimeter is tripled.

(b) If you triple the lengths of all the sides in a rectangle, what is the change in the area?

Let the rectangle have dimensions x by y, then the area is $A = xy$. If you now triple the sides, the area is $A = (3x)(3y) = 9xy$, so we see that the area is multiplied by nine.

7. Max interpreted 5 cm^2 as shown below. Explain to Max why he is incorrect, and show him how to correctly interpret 5 cm^2.



5 cm^2 means we have 5 squares that are 1 cm by 1cm. The picture Max drew has 25 squares that are 1 cm by 1cm. An example of a rectangle that represents 5 cm^2 is below.