HW #1 – ch. 1 problems 7-10,16 – SOLUTIONS

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7. (a) Find 4 rational numbers between $\frac{7}{9}$ and $\frac{8}{9}$. Be sure to all work clearly.

$$\frac{7}{9} = \frac{7\cdot 5}{9\cdot 5} = \frac{35}{45}$$
$$\frac{8}{9} = \frac{8\cdot 5}{9\cdot 5} = \frac{40}{45}$$

Therefore $\boxed{\frac{36}{45}, \frac{37}{45}, \frac{38}{45} \text{ and } \frac{39}{45}}$ are all between $\frac{7}{9}$ and $\frac{8}{9}$.

(b) Find 4 rational numbers between $\frac{a}{c}$ and $\frac{b}{c}$. Be sure to all work clearly.

$$\frac{a}{c} = \frac{a\cdot 5}{c\cdot 5} = \frac{5a}{5c}$$
$$\frac{b}{c} = \frac{b\cdot 5}{c\cdot 5} = \frac{5b}{5c}$$

I claim the four numbers between $\frac{a}{c}$ and $\frac{b}{c}$ are the following.

$$\boxed{\frac{5a+1}{5c}, \frac{5a+2}{5c}, \frac{5a+3}{5c}, \frac{5a+4}{5c}}$$

We need to make sure that each of the numbers really is between $\frac{a}{c}$ and $\frac{b}{c}$. We assumed that $a < b$, so the difference between a and b is at least 1. We used multiplier 5, so the difference in the new numerators will be at least 5. Since the difference in the new numerators is at least five, we know that there are at least four integers between $5a$ and $5b$, so by taking the next four, we know that the last fraction will still be less than $\frac{5b}{5c}$.

(c) Find 4 rational numbers between $\frac{5}{6}$ and $\frac{7}{8}$. Be sure to all work clearly.

$$\frac{5}{6} = \frac{5\cdot 4}{6\cdot 4} = \frac{20}{24} = \frac{20\cdot 5}{24\cdot 5} = \frac{100}{120}$$
$$\frac{7}{8} = \frac{7\cdot 3}{8\cdot 3} = \frac{21}{24} = \frac{21\cdot 5}{24\cdot 5} = \frac{105}{120}$$

Therefore $\boxed{\frac{101}{120}, \frac{102}{120}, \frac{103}{120} \text{ and } \frac{104}{120}}$ are all between $\frac{5}{6}$ and $\frac{7}{8}$.

(d) Find n rational numbers between $\frac{5}{6}$ and $\frac{7}{8}$. Be sure to show all work clearly.

$$\frac{5}{6} = \frac{5\cdot 4}{6\cdot 4} = \frac{20}{24} = \frac{20\cdot(n+1)}{24\cdot(n+1)} = \frac{20n+20}{24n+24}$$
$$\frac{7}{8} = \frac{7\cdot 3}{8\cdot 3} = \frac{21}{24} = \frac{21\cdot(n+1)}{24\cdot(n+1)} = \frac{21n+21}{24n+24}$$

The next integer after $20n+20$ is $20n+20+1$, then $20n+20+2$, then $20n+20+3$ etc. If we continue this until we get n numbers the last one would be $20n+20+n$. However, $20n+20+n = 21n+20$ which is clearly less than $21n+21$. Therefore the n numbers between $\frac{5}{6}$ and $\frac{7}{8}$ are given below.

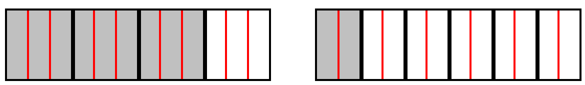
$$\boxed{\frac{20n+21}{24n+24}, \frac{20n+22}{24n+24}, \frac{20n+23}{24n+24}, \ldots, \frac{20n+20+n}{24n+24}}$$

8. In order to add two fractions why do the two fractions need to have a common denominator?

If we think of fractions as pieces of a whole, then addition is simply counting up how many of these pieces we have. However, all of the pieces must be the same size, which means we must get a common denominator.

9. (a) Use a parts of the whole model to find the sum $\dfrac{3}{4} + \dfrac{1}{6}$.

We first represent $\frac{3}{4}$ and $\frac{1}{6}$. We need to get the same size pieces, so we break each piece in the $\frac{3}{4}$ representation into three equal pieces and the $\frac{1}{6}$ representation into two equal pieces. This is shown in red below.



Now all the pieces are the same size. $\frac{3}{4}$ is represented by 9 pieces and $\frac{1}{6}$ is represented by 2 pieces, which gives a total of 11 pieces. We also know there are 12 pieces in a whole.
Therefore $\dfrac{3}{4} + \dfrac{1}{6} = \dfrac{11}{12}$.

(b) Use algebraic properties to find the sum $\dfrac{3}{4} + \dfrac{1}{6}$. Be sure to justify each equality.

$$\frac{3}{4} + \frac{1}{6} \stackrel{i}{=} \frac{3 \cdot 3}{4 \cdot 3} + \frac{1 \cdot 2}{6 \cdot 2} \stackrel{ii}{=} \frac{9}{12} + \frac{2}{12} \stackrel{iii}{=} \frac{11}{12}$$

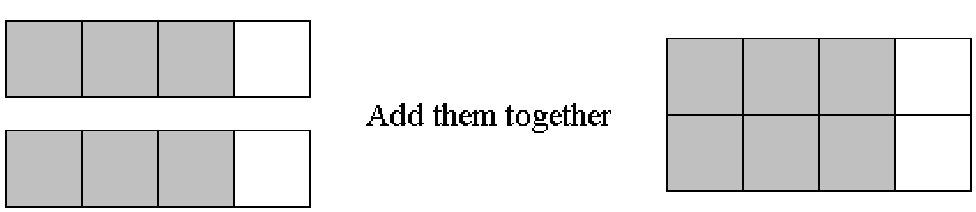
The justifications for the equalities are as follows:

i. Fundamental Law of Fractions
ii. Multiplication in $\mathbb{Z}$
iii. Definition of $+$ in $\mathbb{Q}$.

(c) Describe where the steps in the algebraic method show up in your parts of the whole model.

The multiplier 3 in the fundamental law of fractions, showed up in the parts of the whole model when we broke each of the $\frac{1}{4}$ pieces into 3 equal pieces. Similarly, the multiplier 2 showed up when we broke each of the $\frac{1}{6}$ pieces into 2 equal pieces. Multiplication in $\mathbb{Z}$ showed up when we counted the number of the new size pieces in each representation. Lastly the definition of $+$ in $\mathbb{Q}$ showed up in the parts of the whole model when we counted up the number of shaded pieces all together.

10. Max says that $\dfrac{3}{4} + \dfrac{3}{4} = \dfrac{6}{8}$ and uses this drawing to support his claim:



(a) What does Max understand well about fractions?

Max understands that the fraction $\frac{3}{4}$ is represented by taking a whole, breaking it into 4 equal pieces and shade 3 of them. In other words, he understands the role of the numerator and denominator in a fraction.

(b) What misconception(s) is(are) leading Max to believe that $\dfrac{3}{4} + \dfrac{3}{4}$ can equal $\dfrac{6}{8}$.

Max believes that the whole changes when we add fractions, but this is not true.

(c) How would you respond to Max?

I would remind Max that the whole doesn't change. So we do have 6 pieces, but there are only 4 pieces in a whole. Thus $\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$.

16. Two thirds of the people in a conference room are men. Nobody leaves the room, but 10 more men and 10 more women enter the room.

(a) After the men and women come in are there more men or more women in the room. Explain how you know.

There are more men in the room initially and when we add ten more men and ten more women the difference in the number of women and men does not change. So there are still more men.

(b) After the men and women come in is it still true that two thirds of the people in the room are men? If so, explain why. If not, give a counterexample.

The fraction of men will no longer be $\frac{2}{3}$. Here's an example. Suppose there are 30 people in the room initially. So there are 20 men and 10 women. After the 10 more men and women come in, there will now be 30 men and 20 women. So 30 out of 50 are men, but $\frac{30}{50} = \frac{3}{5}$. So the fraction of men in the room is $\frac{3}{5}$ not $\frac{2}{3}$.