HW #1 – ch. 1 problems 11-15 – SOLUTIONS

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13. You give Max the problem $2\frac{3}{4}+1\frac{1}{5}$. Max does the problem in the following way.

    $2+1=3$
    $\frac{3}{4}+\frac{1}{5}=\frac{15}{20}+\frac{4}{20}=\frac{19}{20}$
    Therefore Max concludes that $2\frac{3}{4}+1\frac{1}{5}=3\frac{19}{20}$.

    (a) Is Max's method and answer correct? If yes, why does it work? If not, explain why not.

    Max's method and answer are correct. He is just using the associative and commutative properties of addition, as you can see below.

    $$2\frac{3}{4}+1\frac{1}{5}=\left(2+\frac{3}{4}\right)+\left(1+\frac{1}{5}\right)=(2+1)+\left(\frac{3}{4}+\frac{1}{5}\right)=3\frac{19}{20}$$

    (b) You then give Max the problem $2\frac{3}{4}+1\frac{2}{5}$. He tries to do it the same way, but gets confused. What could be confusing Max? What can you say to help him?

    He is probably confused because the "fraction part" he gets is bigger than 1. I would tell him his method still works, he just needs to carry over the whole numbers, which I have shown below.
    $2+1=3$
    $\frac{3}{4}+\frac{2}{5}=\frac{15}{20}+\frac{8}{20}=\frac{23}{20}=1\frac{3}{20}$
    So $2\frac{3}{4}+1\frac{2}{5}=3+1\frac{3}{20}=4\frac{3}{20}$.

14. You now give Max the problem $-2\frac{3}{4}+1\frac{1}{5}$. Max does the problem in the following way.

    $-2+1=-1$
    $\frac{3}{4}+\frac{1}{5}=\frac{15}{20}+\frac{4}{20}=\frac{19}{20}$
    Therefore Max concludes that $-2\frac{3}{4}+1\frac{1}{5}=-1\frac{19}{20}$.

    Is Max's method and answer correct? If yes, why does it work? If not, explain why not and give the correct answer.

    Neither Max's answer nor method are correct because $-2\frac{3}{4}$ does not mean $-2+\frac{3}{4}$, but in fact means $-\left(2+\frac{3}{4}\right)$ or in other words it means $-2-\frac{3}{4}$. Therefore I would show Max how to do the problem correctly.

    $-2+1=-1$
    $-\frac{3}{4}+\frac{1}{5}=-\frac{15}{20}+\frac{4}{20}=-\frac{11}{20}$
    Thus Max will get $-2\frac{3}{4}+1\frac{1}{5}=-1+-\frac{11}{20}=-1\frac{11}{20}$.

15. (a) Give the definition of $-\frac{2}{3}$.

    $-\frac{2}{3}$ is the unique number that when added to $\frac{2}{3}$ gives 0.

(b) Prove that $\frac{2}{-3} = -\frac{2}{3}$. (Be sure to justify each equality.)

We know that $-\frac{2}{3}$ is the unique additive inverse of $\frac{2}{3}$. Therefore we need to show that $\frac{2}{-3}$ is the additive inverse of $\frac{2}{3}$ as well. In other words, we need to show $\frac{2}{3} + \frac{2}{-3} = 0$.

$$\frac{2}{-3} + \frac{2}{3} \overset{(i)}{=} \frac{2(-1)}{-3(-1)} + \frac{2}{3} \overset{(ii)}{=} \frac{-2}{3} + \frac{2}{3} \overset{(iii)}{=} \frac{0}{3} \overset{(iv)}{=} 0$$

The justification for the equalities are:

i. Fundamental Law
ii. Multiplication in $\mathbb{Z}$
iii. Addition in $\mathbb{Q}$
iv. Recognizing this fraction as an integer

Thus $\frac{2}{-3}$ is the additive inverse of $\frac{2}{3}$, and so by uniqueness we can conclude that $\frac{2}{-3} = -\frac{2}{3}$.

(c) Prove that $\frac{a}{-b} = -\frac{a}{b}$. (Be sure to justify each equality.)

We know that $-\frac{a}{b}$ is the unique additive inverse of $\frac{a}{b}$. Therefore we need to show that $\frac{a}{-b}$ is the additive inverse of $\frac{a}{b}$ as well. In other words, we need to show $\frac{a}{b} + \frac{a}{-b} = 0$.

$$\frac{a}{-b} + \frac{a}{b} \overset{(i)}{=} \frac{a(-1)}{-b(-1)} + \frac{a}{b} \overset{(ii)}{=} \frac{-a}{b} + \frac{a}{b} \overset{(iii)}{=} \frac{0}{b} \overset{(iv)}{=} 0$$

The justification for the equalities are:

i. Fundamental Law
ii. Multiplication in $\mathbb{Z}$
iii. Addition in $\mathbb{Q}$
iv. Recognizing this fraction as an integer

Thus $\frac{a}{-b}$ is the additive inverse of $\frac{a}{b}$, and so by uniqueness we can conclude that $\frac{a}{-b} = -\frac{a}{b}$.