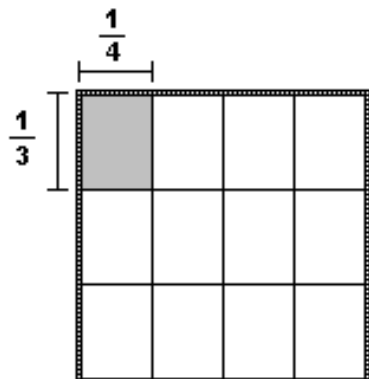


17. Use an area model to illustrate each of the following products of rational numbers.

(a)

$$\frac{1}{3} \cdot \frac{1}{4}$$

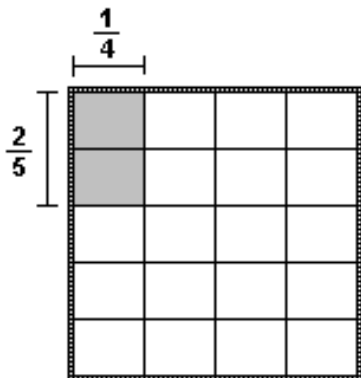


1 shaded piece  
12 pieces in the whole

$$\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$$

(b)

$$\frac{2}{5} \cdot \frac{1}{4}$$

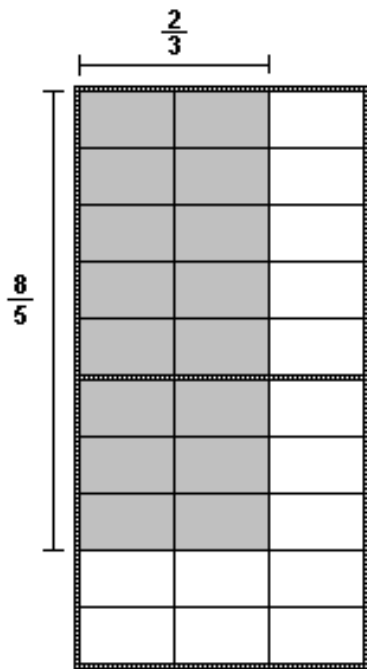


2 shaded pieces  
20 pieces in the whole

$$\frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$$

(c)

$$\frac{8}{5} \cdot \frac{2}{3}$$

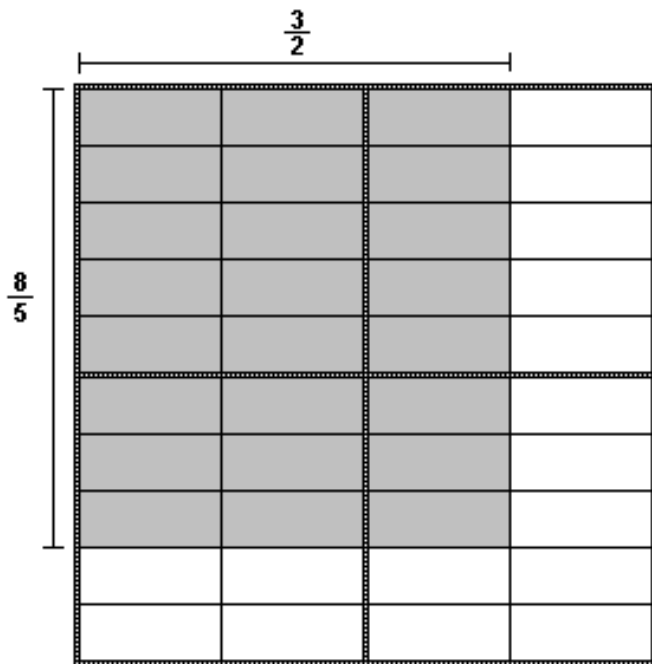


16 shaded pieces  
15 pieces in the whole

$$\frac{8}{5} \cdot \frac{2}{3} = \frac{16}{15}$$

(d)

$$\frac{8}{5} \cdot \frac{3}{2}$$

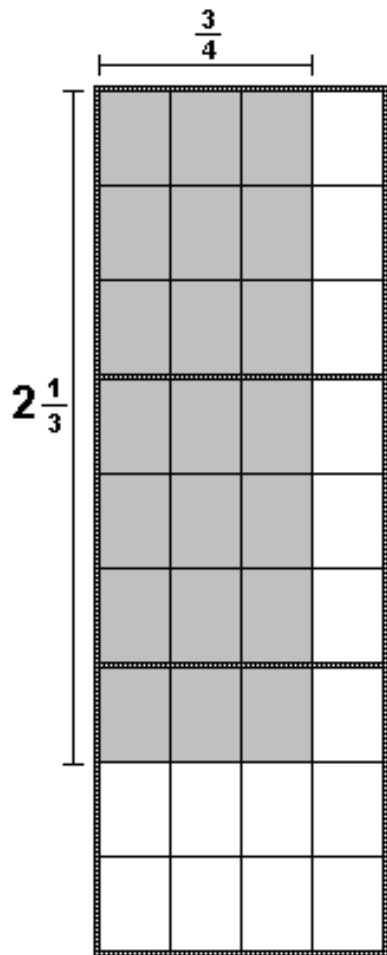


24 shaded pieces  
20 pieces in the whole

$$\frac{8}{5} \cdot \frac{3}{2} = \frac{24}{10}$$

(e)

$$2\frac{1}{3} \cdot \frac{3}{4}$$

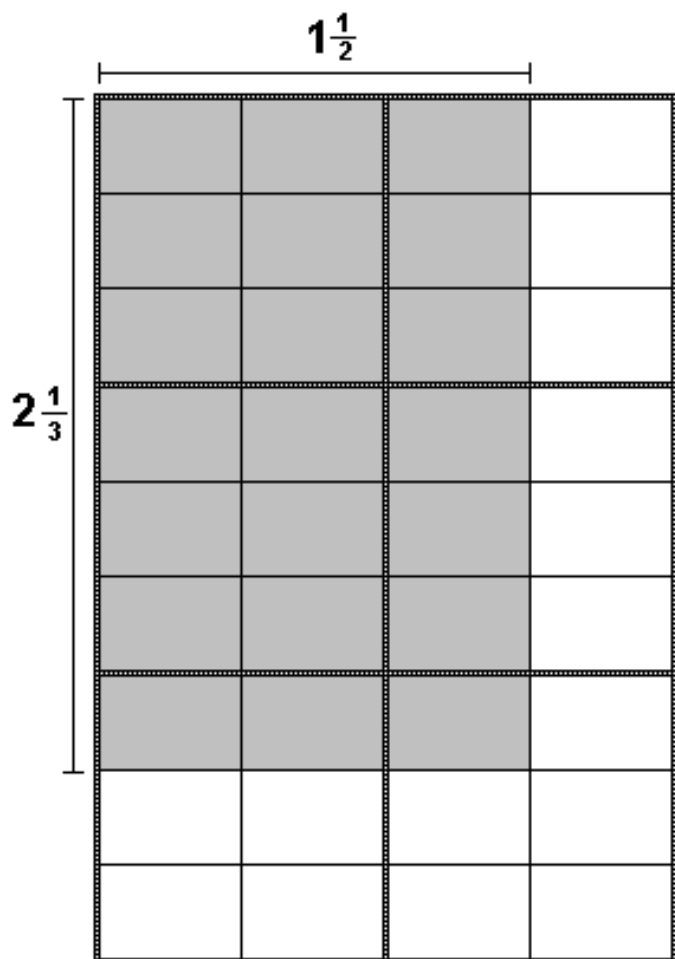


21 shaded pieces  
12 pieces in the whole

$$2\frac{1}{3} \cdot \frac{3}{4} = \frac{21}{12}$$

(f)

$$2\frac{1}{3} \cdot 1\frac{1}{2}$$



21 shaded pieces  
6 pieces in the whole

$$2\frac{1}{3} \cdot 1\frac{1}{2} = \frac{21}{6}$$

22. Suppose  $\frac{a}{b}$  is a positive rational number.

- Choose a positive number that is smaller than 1. If you square your number is the result bigger or smaller than your original number?
- Choose a positive number that is larger than 10 and not an integer. If you square your number is the result bigger or smaller than your original number?
- State a conjecture that indicates which is bigger  $\frac{a}{b}$  or  $\left(\frac{a}{b}\right)^2$ .  
(Note: you may need to break this down into different cases depending on what  $\frac{a}{b}$  is.)

If  $\frac{a}{b}$  is between 0 and 1, then  $\left(\frac{a}{b}\right)^2$  is less than  $\frac{a}{b}$ .

If  $\frac{a}{b}$  is greater than 1, then  $\left(\frac{a}{b}\right)^2$  is greater than  $\frac{a}{b}$ .

- Prove your conjecture.

$$\left(\frac{a}{b}\right)^2 = \frac{a}{b} \cdot \frac{a}{b} = \frac{a^2}{b^2}$$

$$\frac{a}{b} = \frac{ab}{bb} = \frac{ab}{b^2}$$

If  $\frac{a}{b}$  is between 0 and 1, then we know  $a$  is less than  $b$ . So  $a \cdot a$  is less than  $a \cdot b$ . Thus  $a^2$  is less than  $ab$ . Therefore  $\frac{a^2}{b^2}$  is less than  $\frac{ab}{b^2}$ . Hence  $\left(\frac{a}{b}\right)^2$  is less than  $\frac{a}{b}$ .

If  $\frac{a}{b}$  is greater than 1, then we know  $a$  is greater than  $b$ . So  $a \cdot a$  is greater than  $a \cdot b$ . Thus  $a^2$  is greater than  $ab$ . Therefore  $\frac{a^2}{b^2}$  is greater than  $\frac{ab}{b^2}$ . Hence  $\left(\frac{a}{b}\right)^2$  is greater than  $\frac{a}{b}$ .