

HW #6 – ch. 2 problem #1,2,3,7,8,9,10 – SOLUTIONS

1. Use the definition of decimals to express the following numbers as a fraction in simplest form.

$$(a) \quad .36 = \frac{3}{10} + \frac{6}{100} = \frac{30}{100} + \frac{6}{100} = \frac{36}{100} = \frac{9 \cdot 4}{25 \cdot 4} = \boxed{\frac{9}{25}}$$

(b)

$$\begin{aligned} .02918 &= \frac{2}{100} + \frac{9}{1000} + \frac{1}{10000} + \frac{8}{100000} \\ &= \frac{2000}{100000} + \frac{900}{100000} + \frac{10}{100000} + \frac{8}{100000} \\ &= \frac{2918}{100000} \\ &= \frac{1459 \cdot 2}{50000 \cdot 2} \\ &= \boxed{\frac{1459}{50000}} \end{aligned}$$

2. Use multiplication by powers of ten to express the following numbers as a fraction in simplest form.

$$(a) \quad .36 = 36 \cdot 10^{-2} = 36 \cdot \frac{1}{10^2} = \frac{36}{100} = \boxed{\frac{9}{25}}$$

$$(b) \quad .02918 = 2918 \cdot 10^{-5} = 2918 \cdot \frac{1}{10^5} = \frac{2918}{100000} = \boxed{\frac{1459}{50000}}$$

3. Without dividing, express the following rational numbers as a decimal. Be sure to show all steps.

(a) $\frac{3}{4}$

$$\frac{3}{4} = \frac{3 \cdot 25}{4 \cdot 25} = \frac{75}{100} = 75 \cdot 10^{-2} = \boxed{.75}$$

(b) $\frac{9}{40}$

$$\begin{aligned} \frac{9}{40} &= \frac{9 \cdot 25}{40 \cdot 25} \\ &= \frac{225}{1000} \\ &= 225 \cdot 10^{-3} \\ &= \boxed{.225} \end{aligned}$$

7. The definition of .123 is one-tenth plus two hundredths plus three thousandths. However Max hears you call .123 one hundred twenty three thousandths. Explain to Max why you call it that.

When we convert .123 to a fraction we get $\frac{123}{1000}$ and of course the way we say this fraction is “One hundred twenty three thousandths”. The reason we call the fraction (and the decimal) that is because we do actually have 123 pieces each of size one thousandth.

8. Max says to you that he thinks $.36 > .4$ because $36 > 4$. How would you respond to him?

I would tell Max to think of decimals as fractions. So $.36 = \frac{36}{100}$ and $.4 = \frac{4}{10} = \frac{40}{100}$. With fractions once we get a common denominator we need only compare the numerator, so since $36 < 40$ we know $\frac{36}{100} < \frac{40}{100}$ and thus $.36 < .4$.

I would point out to Max that with fractions, we needed to get a common denominator to compare them. What this corresponds to with decimals, is to write the decimals to the same place value. In other words, don't compare .36 and .4, but compare .36 and .40 since $.40 = .4$. Then we can see that $36 < 40$, so $.36 < .40$ and therefore $.36 < .4$.

9. Max hears you say that .3 is the same as .30. This confuses him however, because 3 is not the same as 30. He asks why putting zeros at the end of a decimal doesn't change the number, but putting zeros at the end of a whole number does? How would you respond?

When you put a zero after .3 you do not change the place value of the three, thus you are not changing the number. However, when you put a 0 after 3, you move the 3 from the ones place to the tens place, thus getting a new number.

In general putting a zero at the end of a decimal, does not change the place value of the other digits. However, putting a zero after a whole number changes the place value of all of the other digits, thus changing the number.

10. In class we made the following statement:

Let $\frac{a}{b}$ be a rational number in simplest form. Then $\frac{a}{b}$ can be written as a terminating decimal if and only if the prime factorization of b contains no primes other than 2 and 5.

If we remove the assumption that the fraction is in simplest form, then the statement is false. Give an example to show this.

Consider the fraction $\frac{3}{15}$. If we look at the denominator, the prime factorization is $3 \cdot 5$. Therefore since there is something other than 2 and/or 5 in the prime factorization, we should conclude that this fraction will be represented as a non-terminating decimal, which is incorrect. The conclusion is incorrect because $\frac{3}{15} = \frac{1}{5} = .2$, which is a terminating decimal. Therefore we must have the line that states the fraction must be reduced in order to use this statement.