HW #9 – ch. 3 problem #1, 4-8 – SOLUTIONS

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1. Determine which of the following are rational numbers. Be sure to show all work.

   (a) $\sqrt{\frac{144}{169}} = \frac{\sqrt{144}}{\sqrt{169}} = \frac{12}{13}$ which is a rational number.

   (b) $\sqrt{175}$

   $$175 = 5^2 \cdot 7$$

   Since 175 has an odd number of sevens in the prime factorization, $\sqrt{175}$ is not rational.

   (c) $\sqrt{.\overline{25}}$

   $$\begin{aligned}
   .\overline{25} &= .2525252525\ldots \\
   100(.\overline{25}) &= 25.2525252525\ldots \\
   100(.\overline{25}) - .\overline{25} &= 25.2525252525\ldots - .2525252525\ldots \\
   99(.\overline{25}) &= 25 \\
   .\overline{25} &= \frac{25}{99}
   \end{aligned}$$

   $$\sqrt{.\overline{25}} = \sqrt{\frac{25}{99}} = \frac{\sqrt{25}}{\sqrt{99}} = \frac{5}{\sqrt{99}}$$

   $99 = 3^2 \cdot 11$, since 11 only shows up once, we know $\sqrt{99}$ is irrational. We could prove (like problem 1) that $\frac{5}{\sqrt{99}}$ is irrational. Thus $\sqrt{.\overline{25}}$ is irrational.

   (d) $\sqrt{1.\overline{7}}$

   $$\begin{aligned}
   1.\overline{7} &= 1.77777\ldots \\
   10(1.\overline{7}) &= 17.77777\ldots \\
   10(1.\overline{7}) - 1.\overline{7} &= 17.77777\ldots - 1.77777\ldots \\
   9(1.\overline{7}) &= 16 \\
   1.\overline{7} &= \frac{16}{9}
   \end{aligned}$$

   $$\sqrt{1.\overline{7}} = \sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

   Thus $\sqrt{1.\overline{7}}$ is rational.

4. Ms. Frank (one of your colleagues) uses her calculator to compute $\sqrt{13}$. The calculator spits out the answer 3.60555.

   (a) Ms. Frank proceeds to tell her class that $\sqrt{13} = 3.60555$. Explain to Ms. Frank why $\sqrt{13}$ cannot possibly be represented as a terminating decimal.

   $\sqrt{13}$ is an irrational number, however 3.60555 is a rational number since it is a terminating decimal. Therefore $\sqrt{13} \neq 3.60555$ because an irrational cannot equal a rational.

   (b) After your explanation above, Ms. Frank then concludes that $\sqrt{13} = 3.60\overline{5}$. Explain to Ms. Frank why $\sqrt{13}$ cannot possibly be represented as a repeating decimal.

   $\sqrt{13}$ is an irrational number, however $3.60\overline{5}$ is a rational number since it is a repeating decimal. Therefore $\sqrt{13} \neq 3.60\overline{5}$ because an irrational cannot equal a rational.

5. Give an example for each of the following.
   (a) A natural number. 1
   (b) A whole number that is not a natural number. 0
   (c) An integer that is not a whole number. -2
   (d) A rational number that is not an integer. $\frac{1}{2}$
   (e) A real number that is not a rational number. $\sqrt{2}$

6. Is it possible to answer any of the questions from problem 1(b)-(e) in the reverse? For example, a rational number that is not a real number or an integer that is not a rational number, etc. If yes, give an example. If no, explain why not.

   You cannot answer any of the above in the reverse because a natural number is a whole number which is an integer which is a rational number which is a real number. In other words, $\mathbb{N} \subseteq \mathbb{W} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$.

7. We proved in class that $\sqrt{7}$ is an irrational number. Using this fact, prove the following statements.
   (a) $2 - \sqrt{7}$ is irrational.
   Suppose $2 - \sqrt{7}$ is rational. So there are integers a and b such that

$$\begin{aligned} 2 - \sqrt{7} &= \frac{a}{b} \\ -\sqrt{7} &= \frac{a}{b} - 2 \\ -\sqrt{7} &= \frac{a - 2b}{b} \\ \sqrt{7} &= \frac{-a + 2b}{b} \end{aligned}$$

   Thus we have written $\sqrt{7}$ as an integer over an integer, which of course is impossible since $\sqrt{7}$ is irrational. Thus $2 - \sqrt{7}$ must be irrational.
   (b) $6\sqrt{7}$ is irrational.
   Suppose $6\sqrt{7}$ is rational. So there are integers a and b such that

$$\begin{aligned} 6\sqrt{7} &= \frac{a}{b} \\ \sqrt{7} &= \frac{a}{6b} \end{aligned}$$

   Thus we have written $\sqrt{7}$ as an integer over an integer, which of course is impossible since $\sqrt{7}$ is irrational. Thus $6\sqrt{7}$ must be irrational.

8. Consider the numbers $.5\overline{3}$ and $.5\overline{4}$.
   (a) Find a rational number between these two numbers. $.534$
   (b) Find an irrational number between these two numbers. $.5343343334\ldots$