Author:_____ **Percents**

Group Members:

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1. Sally and Sam work for Acme Co. They started at the same time with the same salary. Their payroll history for the first two years went as follows.

   - At the end of Sally's first year at Acme Co. she received a 10% raise, and at the end of her second year she received a 10% reduction in salary.
   - At the end of Sam's first year at Acme Co. he received a 10% reduction in salary, and at the end of his second year he received a 10% raise.

   (a) If they both make \$40,000 in their first year. How much will each person be making in their second year?

   $40000 + (.1)40000 = 44,000$

   $40000 - (.1)40000 = 36,000$

   In the second year Sally will make \$44,000 and Sam will make \$36,000.

   (b) How much will each person be making in their third year?

   $44000 - (.1)44000 = 39,600$

   $36000 + (.1)36000 = 39,600$

   During the third year they will both be making \$39,600.

   (c) Which option is better, Sally's or Sam's? Why?
   Sally and Sam are making the same in their first year and third year, but Sally is making more in the second year, so Sally's option is better.

   (d) Suppose you don't know their starting salary. Will Sally and Sam have the same salary in their third year? How do you know?
   Let $x =$ their starting salary.
   Sally's salary during the second year $= 1.1x$ and Sally's salary during the third year is $.9(1.1x)$.
   Sam's salary during the second year $= .9x$ and Sam's salary during the third year is $1.1(.9x)$.
   Therefore by the commutative property of multiplication, they are both making the same amount in the third year.

   (e) Suppose you don't know their starting salary. Which option is better, Sally's or Sam's? Why?
   Sally's total for the first three years is $x + 1.1x + .9(1.1x) = 3.09x$
   Sam's total for the first three years is $x + .9x + 1.1(.9x) = 2.89x$
   Thus during the first three years Sam will make less overall, so Sally's option is better.

2. For each of the following problems explain why the method used is correct.

   (a) Joe is in Target and he sees a shirt he wants that costs \$12. He notices that it is on sale for 25% off, so he computes the amount he will save in his head by dividing 12 by 4. Thus he will get \$3 dollars off.

   $$25\% \text{ of } 12 = \frac{25}{100} \cdot 12 = \frac{1}{4} \cdot 12 = 12 \div 4$$

   So this method works because 25% is equal to $\frac{1}{4}$, and multiplying by $\frac{1}{4}$ is the same as dividing by 4.

(b) Joe now wants to buy a CD player that costs \$150 dollars, but it is on sale for 20% off. In his head he finds 20% of 100 and 20% of 50 and gets 20 and 10 respectively. Thus he will save \$30 on the cd player.

20% of $150 = .20(150) = .20(100 + 50) = .20(100) + .20(50)$

So this method works by the distributive property of multiplication over addition.

(c) Joe is almost done shopping and he sees The Wiggles' Greatest Hits CD which costs \$24. However it is on sale for 15% off. He finds 10% of 24, which is 2.4 and then takes half of that, which is 1.2. Thus he will save \$3.60 on the CD.

$$.15(24) = (.10 + .05)(24) = .10(24) + .05(24) = .10(24) + \frac{(.10)}{2} \cdot (24) = .10(24) + \frac{.10(24)}{2}$$

So first we use the distributive property of multiplication over addition to split up the 15%. We then use the fact that 5% is half of 10%.

(d) Joe is in Target again, but this time he has his calculator. It is a good thing because he sees a pair of pants that cost \$24.99 and they are on sale for 22% off. So he punches .78 times 24.99 in his calculator and gets 19.4922. Thus the shirt is on sale for \$19.49.

$24.99 - .22(24.99) = (1 - .22)(24.99) = .78(24.99)$

So this method works by the distributive property of multiplication over addition.

(e) Joe wants to buy a book that is \$18.95. Sales tax in his county is 7.25%. To find the total price he punches 1.0725×18.95 into his calculator and gets \$20.32.

$18.95 + 18.95(.0725) = 18.95(1 + .0725) = 18.95(1.0725) = 20.32$

So this method works by the distributive property of multiplication over addition.

3. (a) Is 30% of 40 equal to 40% of 30?

30% of $40 = .30(40) = 12$

40% of $30 = .40(30) = 12$

(b) Is this true in general? In other words, is A% of B equal to B% of A? If so, prove it. If not, find a counterexample.

$$A\% \text{ of } B = \frac{A}{100} \cdot B = \frac{AB}{100}$$

$$B\% \text{ of } A = \frac{B}{100} \cdot A = \frac{BA}{100}$$

Thus by the commutative property of multiplication they are equal.

4. Write a word problem whose answer is 95%.

5. Write a word problem whose answer is 105%.