Author:______________________________ **Prime Factorizations and more...**

Group Members:

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1. Find the prime factorization of the following numbers.

   $24 = 2^3 \cdot 3$ $\quad$ $80 = 2^4 \cdot 5$ $\quad$ $45 = 3^2 \cdot 5$

   $24^2 = 2^6 \cdot 3^2$ $\quad$ $80^2 = 2^8 \cdot 5^2$ $\quad$ $45^2 = 3^4 \cdot 5^2$

   $3 \cdot 24^2 = 2^6 \cdot 3^3$ $\quad$ $3 \cdot 80^2 = 2^8 \cdot 3 \cdot 5^2$ $\quad$ $3 \cdot 45^2 = 3^5 \cdot 5^2$

2. For each of the numbers below, how many 3's are in their prime factorization.

   24 One $\quad$ 80 Zero $\quad$ 45 Two

   24^2 Two $\quad$ 80^2 Zero $\quad$ 45^2 Four

   $3 \cdot 24^2$ Three $\quad$ $3 \cdot 80^2$ One $\quad$ $3 \cdot 45^2$ Five

3. For each of the numbers below, are there an odd or even number of 3's in their prime factorization.

   24 odd $\quad$ 80 even $\quad$ 45 even

   24^2 even $\quad$ 80^2 even $\quad$ 45^2 even

   $3 \cdot 24^2$ odd $\quad$ $3 \cdot 80^2$ odd $\quad$ $3 \cdot 45^2$ odd

4. Notice that all of the squared numbers above have an even amount of threes in their prime factorization. Could you ever find a natural number, a, so that a^2 has an odd amount of threes in its prime factorization? If so, find one. If not, explain why not.

   When we square a number, each of the exponents in the prime factorization are multiplied by 2. In other words, all of the factors show up twice as many times. Thus a^2 will always have an even number of threes in its prime factorization.

5. Does the prime factorization of $3a^2$ always have an even number of threes, always have an odd number of threes or does it depend on what a is?

   From above we know a^2 has an even number of threes in its prime factorization and if we multiply by three, that means we have one more three in the prime factorization. Thus we have "even +1" amount of threes, which means we have an odd number of threes in the prime factorization of $3a^2$.

6. In fact there is nothing special about the number 3 (other than that it is prime). I could have asked you to do the first page of this worksheet but ask the questions about other primes, for example 2 or 5 or 7 etc.

   (a) What can you say about the number of 2's in the prime factorization of a^2?

   Just as we saw with the prime 3, we can see that there will be an even number of twos in the prime factorization of a^2.

   (b) What can you say about the number of 5's in the prime factorization of a^2?

   Just as we saw with the prime 3, we can see that there will be an even number of five's in the prime factorization of a^2.

   (c) What can you say about the number of 7's in the prime factorization of a^2?

   Just as we saw with the prime 3, we can see that there will be an even number of seven's in the prime factorization of a^2.

   (d) What can you say about the number of any prime in the prime factorization of a^2?

   Just as we saw with the prime 3, we can see that there will be an even number of each prime in the prime factorization of a^2.

7. (a) Is there an even amount or an odd amount of twos in the prime factorization of $2a^2$?
   There is an even number of twos in the prime factorization of a^2 plus one more two in the prime factorization of 2, so there will be an $\boxed{\text{odd}}$ number of twos in the prime factorization of $2a^2$.

   (b) Is there an even amount or an odd amount of twos in the prime factorization of $12a^2$?
   There is an even number of twos in the prime factorization of a^2 plus two more twos in the prime factorization of 12, so there will be an $\boxed{\text{even}}$ number of twos in the prime factorization of $12a^2$.

   (c) Is there an even amount or an odd amount of threes in the prime factorization of $12a^2$?
   There is an even number of threes in the prime factorization of a^2 plus one more three in the prime factorization of 12, so there will be an $\boxed{\text{odd}}$ number of threes in the prime factorization of $12a^2$.

   (d) Is there an even amount or an odd amount of fives in the prime factorization of $35a^2$?
   There is an even number of fives in the prime factorization of a^2 plus one more five in the prime factorization of 35, so there will be an $\boxed{\text{odd}}$ number of fives in the prime factorization of $35a^2$.

   (e) Is there an even amount or an odd amount of sevens in the prime factorization of $35a^2$?
   There is an even number of sevens in the prime factorization of a^2 plus one more seven in the prime factorization of 35, so there will be an $\boxed{\text{odd}}$ number of sevens in the prime factorization of $35a^2$.

(f) Is there an even amount or an odd amount of threes in the prime factorization of $27a^2$?

There is an even number of threes in the prime factorization of a^2 plus three more threes in the prime factorization of 27, so there will be an $\boxed{\text{odd}}$ number of threes in the prime factorization of $27a^2$.

(g) Is there an even amount or an odd amount of threes in the prime factorization of $9a^2$?

There is an even number of threes in the prime factorization of a^2 plus two more threes in the prime factorization of 9, so there will be an $\boxed{\text{even}}$ number of threes in the prime factorization of $9a^2$.

8. If possible find natural numbers a and b that satisfy the following equations. If it is not possible, explain why not.

(a) $9b^2 = a^2$

$a = 1, b = 3$

(b) $3b^2 = a^2$

The prime factorization of $3b^2$ has an odd amount of threes, but the prime factorization of a^2 has an even amount of threes. Therefore there cannot be any natural numbers a and b such that $3b^2$ is equal to a^2.

(c) $2b^2 = a^2$

The prime factorization of $2b^2$ has an odd amount of twos, but the prime factorization of a^2 has an even amount of twos. Therefore there cannot be any natural numbers a and b such that $2b^2$ is equal to a^2.

(d) $12b^2 = a^2$

The prime factorization of $12b^2$ has an odd amount of threes, but the prime factorization of a^2 has an even amount of threes. Therefore there cannot be any natural numbers a and b such that $12b^2$ is equal to a^2.

(e) $35b^2 = a^2$

The prime factorization of $35b^2$ has an odd amount of fives, but the prime factorization of a^2 has an even amount of fives. Therefore there cannot be any natural numbers a and b such that $35b^2$ is equal to a^2.

(f) $27b^2 = a^2$

The prime factorization of $35b^2$ has an odd amount of threes, but the prime factorization of a^2 has an even amount of threes. Therefore there cannot be any natural numbers a and b such that $35b^2$ is equal to a^2.