

Section 3.5/3.8 homework

1. From the text (section 3.8): # 81, 82, 83
2. Let F be a field of order 8.
 - (a) Prove that $\text{char}(F) = 2$.
 - (b) Suppose $a, b \in F$ with $a^2 + ab + b^2 = 0$.
 - i. Prove $a^3 = b^3$.
 - ii. Prove $a^2 = b^2$.
 - iii. Prove $a = b = 0$.
3. Let R be a commutative ring. Let S be a commutative ring without zero-divisors. Let $\phi : R \rightarrow S$ be a ring homomorphism with $K = \ker \phi$. Prove that for any $a, b \in R$, if $ab \in K$, then $a \in K$ or $b \in K$.
4. Let $A = 2\mathbb{Z}$ and $B = 8\mathbb{Z}$.
 - (a) Show that the group A/B is isomorphic to the group \mathbb{Z}_4 .
 - (b) Show that the ring A/B is not isomorphic to the ring \mathbb{Z}_4 .