Section 3.5 homework

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1. From the text (section 3.5): # 40, 41, 42, 44, 45, 49, 50(i)(ii), 51
2. Recall that an element in a ring is called idempotent if $a^2 = a$. Prove that in a commutative ring of characteristic 2, the set of idempotents forms a subring.
3. Let F be a finite field with n elements. Prove that $x^{n-1} = 1$ for all nonzero x in F.
4. Let R be a ring with unity and let I be an ideal in R.
   (a) Prove $I = R$ if and only if I contains an element of $U(R)$.
   (b) Prove that if R is a field, then R has exactly two ideals.
   (c) Show that a homomorphism from a field onto a ring with more than one element must be an isomorphism.