19. Identify the following expressions as true or false. If true, prove the result; if false, give a counterexample.

(a) $\sum_{i=1}^{n}(a_i+1) = \left(\sum_{i=1}^{n} a_i\right) + n$

Proof.

$$\begin{aligned}\sum_{i=1}^{n}(a_i+1) &= (a_1+1)+(a_2+1)+\cdots+(a_n+1)\\ &= (a_1+a_2+\cdots+a_n)+\underbrace{(1+1+\cdots+1)}_{n\text{ times}}\\ &= \left(\sum_{i=1}^{n} a_i\right)+n\end{aligned}$$

□

(b) $\sum_{i=1}^{n}\left(\sum_{j=1}^{m} 1\right) = mn$

Proof.

$$\begin{aligned}\sum_{i=1}^{n}\left(\sum_{j=1}^{m} 1\right) &= \sum_{i=1}\underbrace{(1+1+\cdots+1)}_{m\text{ times}}\\ &= \sum_{i=1}^{n} m\\ &= \underbrace{m+m+\cdots+m}_{n\text{ times}}\\ &= mn\end{aligned}$$

□

(c) $\sum_{j=1}^{m}\left(\sum_{i=1}^{n} a_i b_j\right) = \left[\sum_{i=1}^{n} a_i\right]\left[\sum_{j=1}^{m} b_j\right]$

Proof.

$$\begin{aligned}\sum_{j=1}^{m}\left(\sum_{i=1}^{n} a_i b_j\right) &= \sum_{j=1}^{m}(a_1b_j+a_2b_j+\cdots+a_nb_j)\\ &= \sum_{j=1}^{m}(a_1+a_2+\cdots+a_n)b_j\\ &= (a_1+a_2+\cdots+a_n)\sum_{j=1}^{m} b_j\\ &= \left(\sum_{i=1}^{n} a_i\right)\left(\sum_{j=1}^{m} b_j\right)\end{aligned}$$

□