6. Let $A = [a_{ij}]$ be the $n \times n$ matrix defined by $a_{ii} = k$ and $a_{ij} = 0$ if $i \neq j$. Show that if $B$ is any $n \times n$ matrix, then $AB = kB$.

   *Proof.* Let $B = [b_{ij}]$. Then $AB = [c_{ij}]$ where $c_{ij} = \sum_{l=1}^{n} a_{il} b_{lj}$. However $a_{il} = 0$ if $i \neq l$ and $a_{ii} = k$. Thus $[c_{ij}] = [a_{ii} b_{ij}] = [k b_{ij}] = kB$. Thus $AB = kB$. □

35. Show that $(A - B)^T = A^T - B^T$.

   *Proof.* $$(A - B)^T = (A + (-1)B)^T = A^T + \Big((-1)B\Big)^T = A^T + (-1)B^T = A^T - B^T$$

   The first equality is by the definition of subtraction of matrices. The second equality is due to Theorem 1.4(b). The third equality is due to Theorem 1.4(d). The fourth equality is due to the definition of subtraction of matrices. □

36. Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be solutions ot the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

   Since $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions we know that $A\mathbf{x}_1 = 0$ and $A\mathbf{x}_2 = 0$.

   (a) Show that $\mathbf{x}_1 + \mathbf{x}_2$ is a solution.
   $A(\mathbf{x}_1 + \mathbf{x}_2) = A\mathbf{x}_1 + A\mathbf{x}_2 = 0 + 0 = 0$
   Thus $\mathbf{x}_1 + \mathbf{x}_2$ is a solution.

   (b) Show that $\mathbf{x}_1 - \mathbf{x}_2$ is a solution.
   $A(\mathbf{x}_1 - \mathbf{x}_2) = A\mathbf{x}_1 - A\mathbf{x}_2 = 0 - 0 = 0$
   Thus $\mathbf{x}_1 - \mathbf{x}_2$ is a solution.

   (c) For any scalar $r$, show that $r\mathbf{x}_1$ is a solution.
   $A(r\mathbf{x}_1) = r(A\mathbf{x}_1) = r(0) = 0$
   Thus $r\mathbf{x}_1$ is a solution.

   (d) For any scalars $r$ and $s$, show that $r\mathbf{x}_1 + s\mathbf{x}_2$ is a solution.
   $A(r\mathbf{x}_1 + s\mathbf{x}_1) = r(A\mathbf{x}_1) + s(A\mathbf{x}_1) = r(0) + s(0) = 0$
   Thus $r\mathbf{x}_1 + s\mathbf{x}_1$ is a solution.