16. Give a geometric description of the matrix transformation $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(\mathbf{u}) = A\mathbf{u}$ for each given matrix A.

    (a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

    $$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ a \end{bmatrix}$$

    The resulting vector is a reflection through the line $y = x$.

    (b) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

    $$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -b \\ -a \end{bmatrix}$$

    The resulting vector is a reflection through the line $y = -x$.

18. Find vectors $\mathbf{u}$ and $\mathbf{v}$ not equal so that $f(\mathbf{u}) = f(\mathbf{v}) = \mathbf{w}$.

    (a) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

    $$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

    $$\begin{bmatrix} a + 2b \\ b - c \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

    $a + 2b = 0$ impies $a = -2b$
    $b - c = -1$ implies $b = c - 1$

    Choosing two values for c we can find two vectors. Let $c = 0$ and $c = 1$, then $u = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

    (b) $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

    $$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

    $$\begin{bmatrix} 2a + b \\ 2b - c \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

    $2a + b = 4$ implies $b = 4 - 2a$
    $2b - c = 4$ implies $c = 2b - 4$

    Choosing two values for a we can find two vectors. Let $a = 0$ and $a = 1$, then $u = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.