10. Find a 2×1 matrix $\mathbf{x}$ with entries not all zero such that $A\mathbf{x} = 4\mathbf{x}$, where $A = \begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix}$.

$$\begin{aligned} A\mathbf{x} &= 4\mathbf{x} \\ \begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= 4\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ \begin{bmatrix} 4x_1 + x_2 \\ 2x_2 \end{bmatrix} &= \begin{bmatrix} 4x_1 \\ 4x_2 \end{bmatrix} \end{aligned}$$

$4x_1 + x_2 = 4x_1$ implies $x_2 = 0$. Moreover, if $x_2 = 0$, then x_1 can by anything. So one such vector is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

14. In the following linear system, determine all values of a for which the resulting linear system has
    - (a) no solution
    - (b) a unique solution
    - (c) infinitely many solutions

$$\begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2 - 5 & a \end{bmatrix}$$

$$\begin{matrix} \\ R_1 - R_2 \rightarrow R_2 \\ R_1 - R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & 0 & 4 - a^2 & 2 - a \end{bmatrix}$$

(a) We get no solution if we have the (3,3) entry equal zero and the (3,4) entry not equal to zero. Therefore $a^2 - 4 = 0$ and $2 - a \neq 0$, thus $\boxed{a = -2.}$

(b) We get a unique solutions as long as neither the (3,3) entry nor the (3,4) entry are zero. Therefore $\boxed{a \neq \pm 2.}$

(c) We get infinitely many solutions if both the (3,3) and (3,4) entries are zero. Therefore $\boxed{a = 2.}$

26. Find an equation relating a, b and c so that the linear system below is consisten for any values a, b and c that satisfy that equation.

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + 3z &= b \\ 5x + 9y - 6z &= c \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right]$$

$$\begin{array}{r} \\ 2R_1 - R_2 \to R_2 \\ 5R_1 - R_3 \to R_3 \end{array} \left[\begin{array}{cccc} 1 & 2 & -3 & a \\ 0 & 1 & -9 & 2a - b \\ 0 & 1 & -9 & 5a - c \end{array}\right]$$

$$\begin{array}{r} \\ \\ R_2 - R_3 \to R_3 \end{array} \left[\begin{array}{cccc} 1 & 2 & -3 & a \\ 0 & 1 & -9 & 2a - b \\ 0 & 0 & 0 & -3a - b + c \end{array}\right]$$

The only way this system will be consistent is if the (3,4) entry is zero. Thus $-3a - b + c = 0$, or rearranging we get $\boxed{3a + b - c = 0}$.