8. Is $\det(AB) = \det(BA)$? Justify your answer.

   Proof. We can use Theorem 3.9 and the fact that multiplication of real numbers is commutative to get the following.

   $$\det(AB) = \det(A)\det(B) = \det(B)\det(A) = \det(BA)$$

   □

   Note: The above proof works because we are assuming in this chapter that all matrices are square matrices. However, if A and B are not square matrices this result may not be true. For example, let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$. Then we have the following.

   $$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   $$BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

   So $\det(AB) = 1 \neq 0 = \det(BA)$.

14. Show that if $AB = I_n$, then $\det(A) \neq 0$ and $\det(B) \neq 0$.

   Proof. Assume $AB = I_n$. Then we have the following.

   $$\begin{aligned} AB &= I_n \\ \det(AB) &= \det(I_n) \\ \det(A)\det(B) &= 1 \end{aligned}$$

   Since $\det(A)$ and $\det(B)$ are real numbers whose product is non-zero, we can conclude that $\det(A) \neq 0$ and $\det(B) \neq 0$. □

30. Let A be a 3×3 matrix with $\det(A) = 3$.

   (a) What is the reduced row echelon form to which A is row equivalent?
   Since $\det(A) \neq 0$, we know that A is non-singular and hence the reduced row echelon form of A is I_3.

   (b) How many solutions does the homogeneous system $A\mathbf{x} = \mathbf{0}$ have?
   Since A^{-1} exists, we have $\mathbf{x} = A^{-1}\mathbf{0} = \mathbf{0}$. Thus the trivial solution is the only solution.