12. Let W be the set of all 3×3 matrices of the form $\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix}$. Show that W is a subspace of M_{33}.

Proof. Let $\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix}$ and $\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix}$ be in W.

Then $\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix} + \begin{bmatrix} f & 0 & g \\ 0 & h & 0 \\ i & 0 & j \end{bmatrix} = \begin{bmatrix} a+f & 0 & b+g \\ 0 & c+h & 0 \\ d+i & 0 & e+j \end{bmatrix}$. Thus W is closed with respect to matrix addition.

Similarly $r \begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix} = \begin{bmatrix} ra & 0 & rb \\ 0 & rc & 0 \\ rd & 0 & re \end{bmatrix}$. Thus W is closed with respect to scalar multiplication.

Therefore W is a subspace of M_{33}. □

14. Let W be the set of all 2×2 matrices A such that $A\mathbf{z} = \mathbf{0}$, where $\mathbf{z} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Is W a subspace of M_{22}? Explain.

Proof. Let A and B be in W. Then $A\mathbf{z} = \mathbf{0}$ and $B\mathbf{z} = \mathbf{0}$.

Therefore $(A + B)\mathbf{z} = A\mathbf{z} + B\mathbf{z} = \mathbf{0} + \mathbf{0} = \mathbf{0}$. Thus $A + B$ is in W.

Let r be a real number. Then $(rA)\mathbf{z} = r(A\mathbf{z}) = r(\mathbf{0}) = \mathbf{0}$. Thus rA is in W.

Therefore W is a subspace of M_{22}. □