

12. Find a basis for the null space of $A = \begin{bmatrix} 1 & -1 & 2 & 1 & 0 \\ 2 & 0 & 1 & -1 & 3 \\ 5 & -1 & 3 & 0 & 3 \\ 4 & -2 & 5 & 1 & 3 \\ 1 & 3 & -4 & -5 & 6 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 0 \\ 2 & 0 & 1 & -1 & 3 \\ 5 & -1 & 3 & 0 & 3 \\ 4 & -2 & 5 & 1 & 3 \\ 1 & 3 & -4 & -5 & 6 \end{bmatrix}$$

$$\begin{array}{l} 2R_1 - R_2 \rightarrow R_2 \\ 5R_1 - R_3 \rightarrow R_3 \\ 4R_1 - R_4 \rightarrow R_4 \\ R_1 - R_5 \rightarrow R_5 \end{array} \begin{bmatrix} 1 & -1 & 2 & 1 & 0 \\ 0 & -2 & 3 & 3 & -3 \\ 0 & -4 & 7 & 5 & -3 \\ 0 & -2 & 3 & 3 & -3 \\ 0 & -4 & 6 & 6 & -6 \end{bmatrix}$$

$$\begin{array}{l} 2R_2 - R_3 \rightarrow R_3 \\ R_2 - R_4 \rightarrow R_4 \\ 2R_2 - R_5 \rightarrow R_5 \end{array} \begin{bmatrix} 1 & -1 & 2 & 1 & 0 \\ 0 & -2 & 3 & 3 & -3 \\ 0 & 0 & -1 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} R_1 + 2R_3 \rightarrow R_1 \\ R_2 + 3R_3 \rightarrow R_2 \end{array} \begin{bmatrix} 1 & -1 & 0 & 5 & -6 \\ 0 & -2 & 0 & 12 & -12 \\ 0 & 0 & -1 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} 2R_1 - R_2 \rightarrow R_1 \\ -\frac{1}{2}R_2 \end{array} \begin{bmatrix} 2 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -6 & 6 \\ 0 & 0 & -1 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$c = -3d + 3e$$

$$b = 6d - 6e$$

$$a = d$$

Therefore vectors in the Null space are of the form $d \begin{bmatrix} 1 \\ 6 \\ -3 \\ 1 \\ 0 \end{bmatrix} + e \begin{bmatrix} 0 \\ -6 \\ 3 \\ 0 \\ 1 \end{bmatrix}$. Hence $\left\{ \begin{bmatrix} 1 \\ 6 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -6 \\ 3 \\ 0 \\ 1 \end{bmatrix} \right\}$

forms a basis for the Null space.

22. Determine the solution to the linear system $A\mathbf{x} = \mathbf{b}$ and write it in the form $\mathbf{x} = \mathbf{x}_p + \mathbf{x}_h$.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 2 \\ 1 & -3 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & -3 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & 0 & -2 \\ 0 & -2 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_2 = -1$$

$$x_1 = -2x_3$$

$$\mathbf{x} = \begin{bmatrix} -2x_3 \\ -1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$