2. Find a basis for the subspace of P_3 spanned by $S = \{t^3 + t^2 + 2t + 1, t^3 - 3t + 1, t^2 + t + 2, t + 1, t^3 + 1\}$.

   Setting up the coordinate matrix, with the vectors above in the rows and then performing row operations we get:

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & -3 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

   Therefore a basis is $\{t^3 + t^2 + 2t + 1, t^2 + 5t, 2t - 1, 1\}$.

   (Note: We could have kept reducing the matrix, so this is not the only possible basis. In fact if we kept going we would get the basis $\{t^3, t^2, t, 1\}$.)

28. Is $S = \left\{ \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \right\}$ a linearly independent set of vectors in $\mathbb{R}^3$?

   We form the matrix whose rows are the vectors above, so the span of S is the row space of the matrix below. If we can show that the dimension of the row space is three, then we cannot throw out any of the vectors in S and hence they must be linearly independent.

$$\begin{bmatrix} 2 & 2 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$$
$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & -1 & -6 \end{bmatrix}$$
$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & -13 \end{bmatrix}$$

Therefore the rank of A is 3 and hence the set S is a linearly independent set.