

18. Find an orthonormal basis for the solution space of the homogeneous system

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 2 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -5 \\ 0 & -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_2 = 5x_3$$

$$x_1 = -x_2 + x_3 = -4x_3$$

$$\mathbf{x} = x_3 \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$$

Therefore $\left\{ \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix} \right\}$ is a basis for the solution space, but it is not orthonormal. Since there is only one vector we do not need to worry about orthogonal. Therefore we need only make the vector normal. The magnitude of this vector is $\sqrt{16 + 25 + 1} = \sqrt{42}$.

Hence $\left\{ \begin{bmatrix} -\frac{4}{\sqrt{42}} \\ \frac{5}{\sqrt{42}} \\ \frac{1}{\sqrt{42}} \end{bmatrix} \right\}$ is an orthonormal basis for the solution space.