9. Consider the function $L : M_{34} \to M_{24}$ defined by $L(A) = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -3 \end{bmatrix} A$.

   (a) Find $L\left(\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 0 & 2 & 3 \\ 4 & 1 & -2 & 1 \end{bmatrix}\right)$.

$$L\left(\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 0 & 2 & 3 \\ 4 & 1 & -2 & 1 \end{bmatrix}\right) = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 0 & 2 & 3 \\ 4 & 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 15 & 5 & 4 & 8 \\ -5 & -1 & 10 & 2 \end{bmatrix}$$

   (b) Show that L is a linear transformation.
   Let A and B be 3×4 matrices.

$$\begin{aligned} L(A+B) &= \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -3 \end{bmatrix}(A+B) \\ &= \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -3 \end{bmatrix} A + \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -3 \end{bmatrix} B \\ &= L(A) + L(B) \end{aligned}$$

   Therefore $L(A+B) = L(A) + L(B)$.
   Let c be a real number.

$$L(cA) = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -3 \end{bmatrix}(cA) = c\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & -3 \end{bmatrix} A = cL(A)$$

   Therefore $L(cA) = cL(A)$.
   Hence L is a linear transformation.

20. Let $L : P_1 \to P_1$ be a linear transformation for which we know that $L(t+1) = 2t+3$ and $L(t-1) = 3t-2$.

   (a) Find $L(6t-4)$.
   We need to find x and y such that $x(t+1) + y(t-1) = 6t-4$. Therefore $x+y = 6$ and $x - y = -4$. Adding the two equations we get $2x = 2$, so $x = 1$. Therefore $y = 5$.

$$\begin{aligned} L(6t-4) &= L\Big((t+1) + 5(t-1)\Big) \\ &= L(t+1) + 5L(t-1) \\ &= 2t+3+5(3t-2) \\ &= 17t-7 \end{aligned}$$

   (b) Find $L(at+b)$.
   We need to find x and y such that $x(t+1) + y(t-1) = at+b$. Therefore $x+y = a$ and $x - y = b$. Adding the two equations we get $2x = a+b$, so $x = \frac{a+b}{2}$. Therefore $y = \frac{a-b}{2}$.

$$
\begin{aligned}
L(at+b) &= L\Big(\frac{a+b}{2}(t+1) + \frac{a-b}{2}(t-1)\Big) \\
&= \frac{a+b}{2}L(t+1) + \frac{a-b}{2}L(t-1) \\
&= \frac{a+b}{2}(2t+3) + \frac{a-b}{2}(3t-2) \\
&= \left(\frac{5a-b}{2}\right)t + \left(\frac{a+5b}{2}\right)
\end{aligned}
$$