Quiz #3
Math 35
February 16, 2009 **Name:**

---

1. Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$.

$$\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} a+3c & b+3d \\ 5a+2c & 5b+2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{aligned} a+3c &= 1 & b+3d &= 0 \\ 5a+2c &= 0 & 5b+2d &= 1 \\ a &= -\frac{2}{5}c & b &= -3d \\ -\frac{2}{5}c+3c &= 1 & -15d+2d &= 1 \\ c &= \frac{5}{13} & d &= -\frac{1}{13} \\ a &= -\frac{2}{13} & b &= \frac{3}{13} \end{aligned}$$

$$\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{2}{13} & \frac{3}{13} \\ \frac{5}{13} & -\frac{1}{13} \end{bmatrix}$$

2. Find the solution to the matrix equation $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

$$\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{2}{13} & \frac{3}{13} \\ \frac{5}{13} & -\frac{1}{13} \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{6}{13} \\ \frac{11}{13} \end{bmatrix}$$

3. The linear system $AC\mathbf{x} = \mathbf{b}$ is such that A and C are nonsingular with

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \quad C^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \text{and } \mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Find the solution $\mathbf{x}$.

$$\mathbf{x} = C^{-1}A^{-1}\mathbf{b} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 15 \\ 9 \end{bmatrix}$$