

1. Use cofactor expansion to compute the determinant of the matrix  $A$ .

$$A = \begin{bmatrix} 4 & 1 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 2 \end{bmatrix}$$

$$\begin{aligned} \det A &= 3 \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix} + 2 \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} \\ &= 3(6 - 3) + 2(12 - 2) \\ &= 3(3) + 2(10) \\ &= 29 \end{aligned}$$

2. Use the adjoint to compute the inverse of the matrix  $A$ .

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$A_{11} = \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = 3 \qquad A_{21} = - \begin{vmatrix} 2 & 2 \\ 0 & 3 \end{vmatrix} = -6 \qquad A_{31} = \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} = 2$$

$$A_{12} = - \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = 2 \qquad A_{22} = \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = 10 \qquad A_{32} = - \begin{vmatrix} 4 & 2 \\ 0 & 2 \end{vmatrix} = -8$$

$$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \qquad A_{23} = - \begin{vmatrix} 4 & 2 \\ 1 & 0 \end{vmatrix} = 2 \qquad A_{33} = \begin{vmatrix} 4 & 2 \\ 0 & 1 \end{vmatrix} = 4$$

$$\det A = 4(3) + 1(4 - 2) = 12 + 2 = 14$$

$$A^{-1} = \frac{1}{14}(\text{adj}A) = \frac{1}{14} \begin{bmatrix} 3 & -6 & 2 \\ 2 & 10 & -8 \\ -1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{14} & -\frac{3}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{5}{7} & -\frac{4}{7} \\ -\frac{1}{14} & \frac{1}{7} & \frac{2}{7} \end{bmatrix}$$