

1. Determine the components of the vector \overrightarrow{PQ} where $P = (-2, 2, 3)$ and $Q = (-3, 5, 2)$.

$$\overrightarrow{PQ} = \begin{bmatrix} -3 - (-2) \\ 5 - 2 \\ 2 - 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$$

2. Let V be the set of all real numbers; define \oplus by $\mathbf{u} \oplus \mathbf{v} = \mathbf{uv}$ and \odot by $c \odot \mathbf{u} = c + \mathbf{u}$. Is V a vector space?

$$1 \odot (2 \oplus 3) = 1 \odot (2 \cdot 3) = 1 \odot 6 = 1 + 6 = 7$$

$$(1 \odot 2) \oplus (1 \odot 3) = (1 \cdot 2) \oplus (1 \cdot 3) = 2 \oplus 3 = 2 + 3 = 5$$

Thus property 2(a) does not hold and hence V is not a vector space.

3. Let $\mathbf{x} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{z} = \begin{bmatrix} r \\ -1 \\ s \end{bmatrix}$. Find r and s so that $\mathbf{z} - \mathbf{x} = \mathbf{y}$.

$$\begin{bmatrix} r \\ -1 \\ s \end{bmatrix} - \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$$
$$\begin{bmatrix} r - 1 \\ -1 + 2 \\ s - 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$$

Thus $r - 1 = -3$, so $r = -2$. Also, $s - 3 = 3$, so $s = 6$.