Quiz #6
Math 35
April 13, 2009 **Name:**

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1. Find a basis for the subspace of $\mathbb{R}^3$ given by all vectors of the form $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where $b = a + c$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ a+c \\ c \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Therefore $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ spans all of the vectors of the appropriate form. In addition, the vectors in S are linearly independent since they are not scalar multiples of each other. Therefore S is a basis for all vectors of the aforementioned form.

2. Given an example of a two-dimensional subspace of $\mathbb{R}^4$.

There are lots of possibilities here, so I will give just one.

Let S be the set given below.

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Let $V = \text{span } S$. Then V is a two-dimensional subspace of $\mathbb{R}^4$.

3. Find a basis for the solutions space of the homogeneous system $(\lambda I_n - A)\mathbf{x} = \mathbf{0}$ where $\lambda = 1$ and $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$.

$$\lambda I_n - A = \begin{bmatrix} -2 & -2 \\ -1 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix}$$

$$-2x_1 - 2x_2 = 0 \Rightarrow x_1 = -x_2$$

Therefore $\mathbf{x} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and hence $S = \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ is a basis for the solution space.