Quiz #7
Math 35
April 20, 2009 **Name:**

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1. Find a basis for the row space of A where $A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$$

$$\begin{matrix} \\ R_2 - R_1 \to R_1 \\ 3R_1 - R_3 \to R_3 \\ 2R_1 - R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & -2 & 7 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -8 & 24 & -5 \\ 0 & -5 & 15 & -3 \end{bmatrix}$$

$$\begin{matrix} \\ \\ 8R_2 + R_3 \to R_3 \\ 5R_2 + R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & -2 & 7 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\begin{matrix} \\ \\ \\ \frac{3}{5}R_3 - R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & -2 & 7 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \left\{ \begin{bmatrix} 1 & -2 & 7 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \right\}$$

Therefore B is a basis for the row space of A.

2. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$.

   (a) Find the distance between $\mathbf{u}$ and $\mathbf{v}$.

$$||\mathbf{u} - \mathbf{v}|| = \left|\left| \begin{bmatrix} 5 \\ 7 \end{bmatrix} \right|\right| = \sqrt{5^2 + 7^2} = \sqrt{74}$$

(b) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\ ||\mathbf{v}||} = \frac{-14}{\sqrt{5}\sqrt{41}}$$

3. If possible, find a and b so that $\mathbf{v} = \begin{bmatrix} a \\ b \\ 2 \end{bmatrix}$ is orthogonal to both $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

We want $\mathbf{v}\cdot\mathbf{w}$ to equal 0 and $\mathbf{v}\cdot\mathbf{x}$ to equal 0.

$$\mathbf{v}\cdot\mathbf{w} = 0 \Rightarrow a + 2b + 2 = 0$$

$$\mathbf{v}\cdot\mathbf{x} = 0 \Rightarrow a - b + 2 = 0$$

Subtracting the two equations gives $2b = 0$ and hence $\boxed{b = 0.}$ Plugging back in and solving for a gives $\boxed{a = -2.}$