A **real vector space** is a set V with two operations $\oplus$ and $\odot$ such that the following properties hold.

1. If $\mathbf{u}$ and $\mathbf{v}$ are elements in V, then $\mathbf{u} \oplus \mathbf{v}$ is in V. (Closure with respect to $\oplus$)

   (a) $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$ for all $\mathbf{u}, \mathbf{v}$ in V.

   (b) $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$.

   (c) There exists an element $\mathbf{0}$ in V such that $\mathbf{u} \oplus \mathbf{0} = \mathbf{u}$ and $\mathbf{0} \oplus \mathbf{u} = \mathbf{u}$ for any $\mathbf{u}$ in V.

   (d) For each $\mathbf{u}$ in V there exists an element $-\mathbf{u}$ in V such that $\mathbf{u} \oplus -\mathbf{u} = \mathbf{0}$ and $-\mathbf{u} \oplus \mathbf{u} = \mathbf{0}$.

2. If $\mathbf{u}$ is any element in V and c is any real number, then $c \odot \mathbf{u}$ is in V. (Closure with respect to $\odot$)

   (a) $c \odot (\mathbf{u} \oplus \mathbf{v}) = (c \odot \mathbf{u}) \oplus (c \odot \mathbf{v})$ for $\mathbf{u}, \mathbf{v}$ in V and any real number c.

   (b) $(c + d) \odot \mathbf{u} = (c \odot \mathbf{u}) \oplus (d \odot \mathbf{u})$ for any $\mathbf{u}$ in V and any real numbers c and d.

   (c) $c \odot (d \odot \mathbf{u}) = (cd) \odot \mathbf{u}$ for any $\mathbf{u}$ in V and any real numbers c and d.

   (d) $1 \odot \mathbf{u} = \mathbf{u}$ for any $\mathbf{u}$ in V.