1. Let \( p(x) = x^4 + x^3 + x^2 + x + 1 \in \mathbb{Q}[x] \), and let \( \eta \) be a root of \( p(x) \). Prove that the splitting field for \( p(x) \) is \( \mathbb{Q}(\eta) \).

2. Let \( E \) be a field extension of \( F \).
   (a) Prove that \( \text{Gal}(E/F) \) is a subgroup of \( \text{Aut}(E) \).
   (b) Prove that if \( E \) is an extension of \( \mathbb{Q} \), then \( \text{Gal}(E/\mathbb{Q}) = \text{Aut}(E) \).
       (In other words, any automorphism of \( E \) will fix \( \mathbb{Q} \).
   (c) Prove that if \( E \) is an extension of \( \mathbb{F}_p \), then \( \text{Gal}(E/\mathbb{F}_p) = \text{Aut}(E) \).
       (In other words, any automorphism of \( E \) will fix \( \mathbb{F}_p \).

3. Determine \( \text{Gal} \left( \mathbb{Q}(\sqrt{2})/\mathbb{Q}(\sqrt{2}) \right) \).

4. Let \( \sigma_p : \mathbb{F}_p^n \to \mathbb{F}_p^n \) be given by \( \sigma_p(a) = a^p \). This map is called the Frobenius map.
   (a) Prove \( \sigma_p \in \text{Aut}(\mathbb{F}_p^n) \).
   (b) Determine the order of \( \sigma_p \) in \( \text{Aut}(\mathbb{F}_p^n) \).