Section 3.4+ Homework


2. Prove Corollary 3.34.

3. Let $R$ be a commutative ring with unity and let $a, b \in R$. Prove $(a, b) = \{ar + bs | r, s \in R\}$.

4. Prove that every field is a PID.

5. Let $R$ be an integral domain. Prove that for any $a, b \in R$, the following are equivalent.

   (a) $a$ and $b$ are associates
   (b) $a|b$ and $b|a$
   (c) $(a) = (b)$

6. (a) Let $R$ be an integral domain and $p \in R$. Prove that if $p$ is a prime element, then $p$ is irreducible.

   (b) Let $R$ be a PID and $p \in R$. Prove that if $p$ is irreducible, then $p$ is a prime element.

7. Let $R$ be a commutative ring with unity. Let $a, b \in R$ and let $d$ be a GCD of $a$ and $b$. Prove that $ud$ is also a GCD of $a$ and $b$ for every $u \in U(R)$.

8. Let $R$ be a PID and $a, b \in R$. Prove $(a, b) = (d)$ where $d$ is a GCD of $a$ and $b$. 