1. When computing the volume of a solid of revolution using the disk method, we have the formula \[ \int \pi R^2 \, dx. \]

(a) Why is it called the disk method?
We estimate the area of the region by using rectangles, when we rotate those rectangles around the given axis they form disks. Thus the volume of the solid can be estimated by stacked disks. To find the volume exactly we must add up the volume of "infinitely many" disks, which is why we get an integral.

(b) From the definition of integrals we know that \( dx \) comes from \( \Delta x \). What does \( \Delta x \) represent in the disk method?
The height of each of the disks described above is \( \Delta x \).

2. Set up (but do not evaluate) the integral(s) to find the area of the region bounded by \( y = x^2 - 3 \), \( y = 2 - x^2 \) and \( x = 3 \).

\[
x^2 - 3 = 2 - x^2 \\
2x^2 = 5 \\
x^2 = \frac{5}{2} \\
x = \pm \sqrt{\frac{5}{2}}
\]

\[
A = \int_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} (2 - x^2) - (x^2 - 3) \, dx + \int_{\sqrt{\frac{5}{2}}}^{3} (x^2 - 3) - (2 - x^2) \, dx
\]
3. Set up (but do not evaluate) the integral(s) to find the volume of the solid obtained by revolving the region bounded by the given curves about the given line.

(a) \( y = x^2 - 4x + 5 \) and \( y = -x^2 + 6x + 17 \) about \( x \)-axis \hspace{1cm} (The region is shown below.)

\[
x^2 - 4x + 5 = -x^2 + 6x + 17
\]
\[
2x^2 - 10x - 12 = 0
\]
\[
2(x + 1)(x - 6) = 0
\]
\[
x = -1, 6
\]

\[
V = \int_{-1}^{6} \pi \left[ ( -x^2 + 6x + 17)^2 - (x^2 - 4x + 5)^2 \right] \, dx
\]

(b) \( y = -x^2 + 4x - 3 \) and \( y = 0 \) about \( x = 4 \) \hspace{1cm} (The region is shown below.)

\[
V = \int_{1}^{3} 2\pi (4-x)(-x^2 + 4x - 3) \, dx
\]
4. Let \( f(x) = \sin^2 x \). Find the average value of \( f(x) \) on \([0, \frac{\pi}{4}]\).

\[
\text{Avg} = \frac{1}{\frac{\pi}{4}} \int_{0}^{\frac{\pi}{4}} \sin^2 x \, dx \\
= \frac{2}{\pi} \int_{0}^{\frac{\pi}{4}} 1 - \cos 2x \, dx \\
= \frac{2}{\pi} \left[ x - \frac{1}{2} \sin 2x \right]_{0}^{\frac{\pi}{4}} \\
= \frac{2}{\pi} \left( \frac{\pi}{4} - \frac{1}{2} \right) \\
= \frac{1}{2} - \frac{1}{\pi}
\]

Compute the following integrals.

5. 

\[
\int_{1}^{3} \frac{\ln x}{x^3} \, dx \\
u = \ln x \quad dv = \frac{1}{x^3} \, dx \\
du = \frac{1}{x} \, dx \quad v = -\frac{1}{2}x^{-2} \\
= -\frac{1}{2}x^{-2} \ln x \bigg|_{1}^{3} - \int_{1}^{3} -\frac{1}{2}x^{-2} \cdot \frac{1}{x} \, dx \\
= -\frac{\ln x}{2x^2} \bigg|_{1}^{3} + \frac{1}{2} \int_{1}^{3} x^{-3} \, dx \\
= -\frac{\ln 3}{18} + \frac{1}{2} \left( \frac{x^{-2}}{-2} \right) \bigg|_{1}^{3} \\
= -\frac{\ln 3}{18} - \frac{1}{4x^2} \bigg|_{1}^{3} \\
= -\frac{\ln 3}{18} - \frac{1}{36} + \frac{1}{4} \\
= -\frac{\ln 3}{18} + \frac{2}{9}
\]
6. \[
\int \sec^4 x \tan^2 x \, dx = \int \sec^2 x \tan^2 x \sec^2 x \, dx \\
= \int (1 + \tan^2 x) \tan^2 x \sec^2 x \, dx \\
u = \tan x \\
du = \sec^2 x \, dx \\
= \int (1 + u^2)u^2 \, du \\
= \int u^2 + u^4 \, du \\
= \frac{u^3}{3} + \frac{u^5}{5} + C \\
= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C
\]

7. \[
\int x^3e^{x^2} \, dx = \int x^2 e^{x^2} \cdot x \, dx \\
w = x^2 \\
dw = 2x \, dx \\
= \frac{1}{2} \int we^w \, dw \\
u = w \quad dv = e^w \, dw \\
\int du = dv \quad v = e^w \\
= \frac{1}{2} \left[ we^w - \int e^w \, dw \right] \\
= \frac{1}{2} we^w - \frac{1}{2} e^w + C \\
= \frac{1}{2} x^2e^{x^2} - \frac{1}{2} e^{x^2} + C
\]
8. \[
\int \frac{1}{x\sqrt{9 - x^2}} \, dx
\]

\[x = 3 \sin \theta\]

\[dx = 3 \cos \theta \, d\theta\]

\[= \int \frac{1}{3 \sin \theta \sqrt{9 - 9 \sin^2 \theta}} \cdot 3 \cos \theta \, d\theta\]

\[= \int \frac{1}{3 \sin \theta (3 \cos \theta)} \cdot 3 \cos \theta \, d\theta\]

\[= \frac{1}{3} \int \csc \theta \, d\theta\]

\[= -\frac{1}{3} \ln |\csc \theta + \cot \theta| + C\]

\[= -\frac{1}{3} \ln \left| \frac{3}{x} + \frac{\sqrt{9 - x^2}}{x} \right| + C\]