

Name: _____

Group Members: _____

1. All of the following series are geometric. Indicate a and r and determine the interval of convergence, then find the sum of each of the series. (Your answer will be in terms of x).

$$(a) \sum_{n=0}^{\infty} 2 \left(\frac{x}{3}\right)^n = \frac{2}{1 - \frac{x}{3}} = \frac{6}{3 - x}$$

$$a = 2, r = \frac{x}{3}$$

$$(b) \sum_{n=0}^{\infty} (3x)^n = \frac{1}{1 - 3x}$$

$$a = 1, r = 3x$$

$$(c) \sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1 + x^2}$$

$$a = 1, r = -x^2$$

2. Find a power series whose sum is given by the following functions.

$$(a) f(x) = \frac{3}{1+x} = \frac{3}{1-(-x)} = \sum_{n=0}^{\infty} 3(-x)^n = \sum_{n=0}^{\infty} 3(-1)^n x^n$$

$$(b) f(x) = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$(c) f(x) = \frac{x}{1+x^2} = x \cdot \frac{1}{1-(-x)^2} = x \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n+1}$$

$$(d) f(x) = \frac{1}{4+x} = \frac{1 \cdot \frac{1}{4}}{(4+x) \cdot \frac{1}{4}} = \frac{\frac{1}{4}}{1 + \frac{x}{4}} = \frac{\frac{1}{4}}{1 - (-\frac{x}{4})} = \sum_{n=0}^{\infty} \frac{1}{4} \left(-\frac{x}{4}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{4^{n+1}}$$