Name:______________________________ **Volumes of Solids of Revolution**

Group Members:

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1. Consider the region bounded by $y = e^x$, $y = 0$, $x = 0$ and $x = 2$. Set up, but do not evaluate, the integral to find the volume of the solid of revolution obtained by revolving the above region about the given line.

   (a) x-axis

   $$V = \int_0^2 \pi(e^x)^2\, dx$$

   (b) y-axis

   $$V = \int_0^2 2\pi x e^x\, dx$$

   (c) $y = 9$

   $$V = \int_0^2 \pi(9 - e^x)^2\, dx$$

   (d) $x = -2$

   $$V = \int_0^2 2\pi(2 + x)e^x\, dx$$

2. Let R be the region bounded by $y = x^2 - 1$, $y = 0$, $x = 0$ and $x = 2$.

   (a) Use the shell method to find the volume of the solid obtained by revolving R about the y-axis.

   $$\begin{aligned} V &= \int_0^1 2\pi x(-(x^2-1))\, dx + \int_1^2 2\pi x(x^2-1)\, dx \\ &= 2\pi \int_0^1 -x^3 + x\, dx + 2\pi \int_1^2 x^3 - x \\ &= 2\pi \left[-\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1 + 2\pi \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2 \\ &= \frac{\pi}{2} + \frac{9\pi}{2} \\ &= 5\pi \end{aligned}$$

   (b) Use the disk method to find the volume of the solid obtained by revolving R about the y-axis.

   $$\begin{aligned} V &= \int_{-1}^0 \pi(\sqrt{y+1})^2\, dy + \pi(2)^2(3) - \int_0^3 \pi(\sqrt{y+1})^2\, dy \\ &= \pi \int_{-1}^0 y + 1\, dy + 12\pi - \pi \int_0^3 y + 1\, dy \\ &= \pi \left[\frac{y^2}{2} + y\right]_{-1}^0 + 12\pi - \pi \left[\frac{y^2}{2} + y\right]_0^3 \\ &= \frac{\pi}{2} + 12\pi - \frac{15\pi}{2} \\ &= 5\pi \end{aligned}$$

3. Let R be the region bounded by $y = x^2 - 1$, $y = 0$, $x = 0$ and $x = 2$.

(a) Use the shell method to find the volume of the solid obtained by revolving R about the x-axis.

$$\begin{aligned}
V &= \int_{-1}^{0} 2\pi(-y)\sqrt{y+1}\ dy + \int_{0}^{3} 2\pi y(2 - \sqrt{y+1})\ dy \\
&\quad u = y + 1 \\
&\quad du = dy \\
&= -2\pi \int_{0}^{1} (u-1)\sqrt{u}\ du + 2\pi \int_{1}^{4} (u-1)(2-\sqrt{u})\ du \\
&= -2\pi \int_{0}^{1} u^{\frac{3}{2}} - u^{\frac{1}{2}}\ du + 2\pi \int_{1}^{4} 2u - 2 + u^{\frac{1}{2}} - u^{\frac{3}{2}}\ du \\
&= -2\pi \left[\frac{2u^{\frac{5}{2}}}{5} - \frac{2u^{\frac{3}{2}}}{3}\right]_{0}^{1} + 2\pi \left[u^2 - 2u + \frac{2u^{\frac{3}{2}}}{3} - \frac{2u^{\frac{5}{2}}}{5}\right]_{1}^{4} \\
&= \frac{8\pi}{15} + \frac{38\pi}{15} \\
&= \frac{46\pi}{15}
\end{aligned}$$

(b) Use the disk method to find the volume of the solid obtained by revolving R about the x-axis.

$$\begin{aligned}
V &= \int_{0}^{2} \pi(x^2-1)^2\ dx \\
&= \pi \int_{0}^{2} x^4 - 2x^2 + 1\ dx \\
&= \pi \left[\frac{x^5}{5} - \frac{2x^3}{3} + x\right]_{0}^{2} \\
&= \frac{46\pi}{15}
\end{aligned}$$

4. Use calculus to prove that the volume of a right circular cone with height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.

The cone with base radius r and height h can be formed by rotating the line $y = \frac{h}{r}x$ about the y-axis. Therefore we get the following volume.

$$\begin{aligned}
V &= \int_{0}^{h} \pi \left(\frac{r}{h}y\right)^2\ dy \\
&= \frac{\pi r^2}{h^2} \int_{0}^{h} y^2\ dy \\
&= \frac{\pi r^2}{h^2} \left(\frac{y^3}{3}\right)_{0}^{h} \\
&= \frac{\pi r^2}{h^2} \left(\frac{h^3}{3}\right) \\
&= \frac{1}{3}\pi r^2 h
\end{aligned}$$