12. Let $W$ be the set of all $3 \times 3$ matrices of the form \[
\begin{bmatrix}
a & 0 & b \\
0 & c & 0 \\
d & 0 & e
\end{bmatrix}.
\] Show that $W$ is a subspace of $M_{33}$.

Proof. Let \[
\begin{bmatrix}
a & 0 & b \\
0 & c & 0 \\
d & 0 & e
\end{bmatrix}
\] and \[
\begin{bmatrix}
a & 0 & b \\
0 & c & 0 \\
d & 0 & e
\end{bmatrix}
\] be in $W$. Then \[
\begin{bmatrix}
a & 0 & b \\
0 & c & 0 \\
d & 0 & e
\end{bmatrix} + \begin{bmatrix}
f & 0 & g \\
0 & h & 0 \\
i & 0 & j
\end{bmatrix} = \begin{bmatrix}
a + f & 0 & b + g \\
0 & c + h & 0 \\
d + i & 0 & e + j
\end{bmatrix}.
\] Thus $W$ is closed with respect to matrix addition.

Similarly $r \begin{bmatrix}
a & 0 & b \\
0 & c & 0 \\
d & 0 & e
\end{bmatrix} = \begin{bmatrix}
ra & 0 & rb \\
0 & rc & 0 \\
rd & 0 & re
\end{bmatrix}$. Thus $W$ is closed with respect to scalar multiplication.

Therefore $W$ is a subspace of $M_{33}$. \hfill \qed

14. Let $W$ be the set of all $2 \times 2$ matrices $A$ such that $Az = 0$, where $z = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Is $W$ a subspace of $M_{22}$? Explain.

Proof. Let $A$ and $B$ be in $W$. Then $Az = 0$ and $Bz = 0$.

Therefore $(A + B)z = Az + Bz = 0 + 0 = 0$. Thus $A + B$ is in $W$.

Let $r$ be a real number. Then $(rA)z = r(Az) = r(0) = 0$. Thus $rA$ is in $W$.

Therefore $W$ is a subspace of $M_{22}$. \hfill \qed