

Economics 200B
Problem Set 2
Gallet
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Note: Take it as a given that the second-order conditions for utility maximization or expenditure minimization hold. Accordingly, you need not evaluate the second-order conditions.

[1] In our story to date, the consumer takes all prices as given. Instead, consider a consumer of goods x and y for which the price of good x decreases as more of x is consumed. Will this raise or lower the consumer's utility compared to the case for which all prices are fixed? Verify mathematically. Hints: (1) Set up the constrained optimization problem for each scenario (i.e., price of x is fixed and price of x decreases as x increases (i.e., $p_x = f(x)$, with $p'_x < 0$). (2) Evaluate the first-order condition corresponding to the case for which the price of x is not fixed, at the optimum x , y , and λ for which the price of x is fixed. (3) Don't impose a particular functional form for the utility function. Assuming the axioms of consumer preferences hold, argue that your results hold irrespective of the form of the utility function.

[2] Seeger consumes popcorn (C) and butter (B) with utility of: $U(C, B) = \min(2C - B, 2B - C)$.

- A. Placing butter on the horizontal axis, graphically illustrate the income-consumption and price-consumption curves. Based on your graphs, (i) explain whether butter is a normal or an inferior good and (ii) whether popcorn and butter are substitutes or complements. Provide equations of the income-consumption and price-consumption curves.
- B. Determine Seeger's Marshallian and Hicksian demand functions for butter. Based on the Marshallian demand, verify your answers in (A) regarding whether butter is normal or inferior and whether popcorn and butter are substitutes or complements.
- C. Determine the indirect utility and expenditure functions. Verify that Roy's Identity and Shephard's Lemma holds with respect to butter.
- D. Verify the Slutsky equation holds with respect to butter.
- E. Initially, suppose income (I) equals 100, the price of popcorn (P_c) equals 1, and the price of butter (P_b) equals 1. Now, *ceteris paribus*, the price of butter increases to 2.
 - i. Determine the welfare effect on Seeger of this price change by calculating compensating variation (CV). Determine CV , by (1) using the expenditure function and (2) calculating the area under the Hicksian demand curve. Interpret the value of CV .
 - ii. Determine the welfare effect on Seeger of this price change by calculating equivalent variation (EV). Determine EV by (1) using the expenditure function and (2) calculating the area under the Hicksian demand curve. Interpret the value of EV .
- F. Suppose the market for butter consists of 1 million identical consumers, identical in the sense that they all share the same utility function (i.e., $U_i = \min(2C_i - B_i, 2B_i - C_i)$, where i is used to index each consumer, with $i = 1, 2, \dots, 1,000,000$). Currently, each person has an income equal to 100, the price of butter equals 1, and the price of popcorn equals 1. Now, suppose the "We Hate Butter" political wing of Congress is considering a proposal to levy a per unit tax (t) on butter with the goal of raising \$10 million in tax revenue (i.e., \$10 per butter consumer). What tax rate (t) would be required to reach this tax revenue target? Let's suppose that the political feasibility of such a proposal depends on a comparison of the welfare loss borne by butter consumers and the tax revenue raised. In particular, a majority of Congress is unlikely to vote for such a proposal if the welfare loss exceeds the tax revenue. Would Congress likely pass legislation implementing this tax? Does your answer depend on whether CV or EV is used as the measure of welfare loss? Explain.

[3] Consider two consumers, 1 and 2, who each have the following utility function:

$$U_i = x_i y_i + y_i \quad i = 1, 2$$

where x_i is individual i 's consumption of x and y_i is individual i 's consumption of y .

- A. Assuming these are the only two consumers of the good, derive the market (Marshallian) demand for x .
- B. What is the value of the own-price elasticity of market demand at $P_x = 100$, $P_y = 2$, and aggregate income equal \$1000? Classify it as elastic, inelastic, or unit elastic.
- C. Suppose consumer i 's income is \$500, $P_x = 100$, and $P_y = 2$, determine the value of consumer i 's Hicksian price elasticity of demand.
- D. Irrespective of the income and price levels, verify that the Engel Aggregation Condition (with respect to the elasticities) holds for the i th consumer.
- E. Irrespective of the income and price levels, verify that the Homogeneity Condition (with respect to the elasticities) holds for the i th consumer's Marshallian demand for x .