

Economics 200B
Problem Set 4
Spring 2009

Note: The second-order conditions for profit maximization hold for these problems, and so don't be concerned with evaluating the second-order conditions.

[1] Suppose the inverse market demand and cost functions of a single-plant monopolist are given by:

$$\text{Market Demand: } P = 100 - 3Q + 4\sqrt{A}$$

$$\text{Total Cost: } TC = 4Q^2 + 10Q + A, \text{ where } P \text{ is price, } Q \text{ is output, and } A \text{ is advertising expenditure.}$$

Find the values of P, Q, and A that maximize the monopolist's profit.

[2] In Summer 2008, with the price of oil approaching \$150 per barrel, many people argued that oil industry profits should be taxed. Others countered that a tax on profits would merely increase oil prices even further. Reflecting on this, let's consider the impact of a tax on oil prices. For simplicity, we will assume (i) the oil industry is a monopoly, (ii) the inverse market demand for oil is given as $P = P(Q)$, where P is price per barrel and Q is barrels of oil (with $P'(Q) < 0$), and (iii) the total cost to the oil monopoly is given as $TC = TC(Q)$, with $TC'(Q) > 0$.

A. Suppose the government levies an ad valorem tax on profit. Specifically, the oil industry is required to pay the government a percent of the value of its profit. Denoting this percent as δ (with $0 < \delta < 1$) and profit as π , such that the oil industry pays $\delta\pi$ to the government, mathematically argue in favor or against the argument that such an ad valorem tax would increase the price of oil. Hint: You might compare the first-order conditions under the cases of (i) no tax and (ii) tax, and then argue whether or not the tax would affect the optimal quantity (and hence price).

B. Alternatively, suppose the government levies a specific tax on quantity produced. That is, for each barrel produced and sold, the government requires the oil industry to pay \$t, such that the total payment to the government is tQ . Mathematically show whether or not such a tax would increase the price of oil. Hint: You might compare the first-order conditions under the cases of (i) no tax and (ii) tax, and then argue whether or not the tax would affect the optimal quantity (and hence price).

[3] Suppose a monopolist sells toys in catalogs (C) and at retail outlets (R). The demand functions for each are given below:

$$\text{Catalog Demand: } Q_C = 6 - 2P_C$$

$$\text{Retail Demand: } Q_R = 14 - 2P_R$$

The firm's short-run total cost is given as: $STC = 4 + Q$, where $Q = Q_C + Q_R$. Determine the prices and quantities that maximize this firm's profit. Verify that the group charged the higher price has a lower price elasticity (in absolute value).

[4] Consider a representative firm in a monopolistically competitive market. The firm's demand depends on the number of firms in the market. That is, the firm's demand (written in inverse form) is given by: $P = 100 - n - q$, where P is the price charged by the firm, n is the number of firms in the market, and q is the firm's production/sales. Suppose further that the firm's long run total cost is constant at \$100. Determine the number of firms, as well as the price and quantity of this respective firm, coinciding with the long run equilibrium.

[5] Let the inverse market demand and cost functions of a multiplant monopolist be:

$$\text{Market Demand: } P = a - b(q_1 + q_2)$$

$$\text{Plant 1 Total Cost: } TC_1 = \beta_1 q_1^2$$

$$\text{Plant 2 Total Cost: } TC_2 = \beta_2 q_2^2$$

Note: $a, b, \alpha_1, \alpha_2, \beta_1, \beta_2 > 0$.

Assume an exogenous increase in demand increases the value of a , leaving the other parameters unchanged. Mathematically show that output will increase in both plants, with the greater increase for the plant in which marginal cost is increasing (in quantity) less fast.