

Economics 200B
Spring 2009
Gallet
Problem Set 3

[1] Consider a firm that produces with the following production function:

$$Q = 120K^2L^2 - K^3L^3,$$

where Q is the amount produced, K is the amount of the capital input, and L is the amount of the labor input.

Assuming the short run, with capital constant at 1, carefully graph the relationship between Q and L (i.e., the total product curve), MP_L and L (i.e., the marginal product curve), and AP_L and L (i.e., the average product curve). Note: L and Q can never be negative. Be careful to:

- indicate the levels of labor where marginal product is rising and falling.
- explain why this firm should never employ more than 80 workers.

[2] ABC Chemical has the following production function for XYZ, a particular chemical compound:

$$q = (x_1^{1/2} + x_2^{1/2})^2,$$

where q is the amount of XYZ produced, x_1 and x_2 are the respective quantities of the two inputs utilized.

- A. Does the production function exhibit diminishing marginal products? Explain.
- B. Does the marginal rate of technical substitution decline when moving down an isoquant? Explain.
- C. Does the production function exhibit increasing, constant, or decreasing returns to scale? Explain.
- D. Suppose the firm is a price taker, such that the price of XYZ is given at \$20, and the prices of x_1 and x_2 are each given at \$40. Also, suppose in the short run x_2 is fixed at 4. Determine the amount of x_1 that maximizes ABC's short run profit. Make sure to utilize appropriate first- and second-order conditions.
- E. At the output corresponding to the optimal value of x_1 in part D, determine the values of short run total fixed cost, average fixed cost, average variable cost, average total cost, and marginal cost.

[3] A microbrewer desires to produce 120 cases of beer while incurring the least possible cost. The microbrewer's production function is: $Q = 8K^{0.50}L^{0.50}$, and it must pay input prices of \$3 and \$27 per unit, for labor and capital, respectively. What is the optimal combination of K and L ? What is the cost incurred to produce 120 cases of beer?

[4] Suppose a firm operates under constant returns to scale. The firm's production function is indicated by: $Q = f(K, L)$.

- A. Show that Euler's Theorem implies that, under constant returns to scale, $Q = f_K \cdot K + f_L \cdot L$.
- B. Use part A to show if $MP_L > AP_L$, then MP_K must be negative.
- C. Accordingly, should this firm produce where $MP_L > AP_L$? Explain your answer.