Early in the school year, Ms. Harwood posed a mathematical word problem to her fourth-grade students that incorporated several grade-level concepts and skills as outlined in the curriculum standards. She wondered whether her students could draw from their previous knowledge of mathematical skills to determine the answer. What strategies or manipulatives would they use to solve the problem?

On the basis of screening assessments from the beginning of the school year, Harwood organized the class into cooperative groups of four students of mixed abilities. As Harwood walked around the classroom to observe her students, she quickly noticed that eighteen of her twenty-six students understood the word problem, explained and wrote the correct equation, and possessed the skills to multiply two-digit numbers rather quickly—whereas the other students really struggled. Three students appeared to have difficulty setting up the equation, and four students could not multiply two-digit numbers. One student in particular, Matthew, showed signs of frustration and confusion. Harwood immediately asked him to share what he knew about solving the multiplication problem. His explanation for defining multiplication and how to solve the problem of $78 \times 43$ contained some correct information; however, his answer to the problem was incorrect. Matthew knew the answers to some but not all of his basic multiplication facts, and he struggled with addition and place value. As Harwood continued to observe and write notes related to her students’ knowledge and use of the concepts and skills needed to solve this word problem, she recognized that several students would require additional instruction before they could demonstrate mastery.

The scenario above parallels many that take place as we watch our students struggle with certain mathematical skills. One of the most difficult tasks that we face as classroom teachers is finding ways to reach all our students and match each student’s level of mathematical readiness and performance to the skills we are required to teach. In our classrooms and schools, current federal and state requirements have increased the emphasis on accountability for improved
achievement in mathematics for all students (IDEA 2004; NCLB 2002; NCTM 2000). Specifically, the Individuals with Disabilities Education Improvement Act (IDEA) of 2004 outlines Response to Intervention (RtI) as a process for continuously improving achievement for students struggling to learn and to reduce the number of students who are being referred for special education services. The definitions, components, and goals of the recent legislative mandates of RtI and revised curriculum standards in mathematics are about instruction, intervention, and assessments to assure that our students meet the standards and achieve mathematical success. The goal is to ensure that all students learn mathematics through high-quality instruction using evidence-based instructional and intervention methods, products, and practices. On the basis of student assessment results from state, school, and classroom data, teachers and other educators make instructional decisions and problem solve additional teaching strategies to guarantee that each student achieves the instructional goals and standards in mathematics.

What is RtI?
Response to Intervention (RtI) is the practice of (1) providing high-quality instruction or intervention matched to student needs and (2) using learning rate over time and level of performance to (3) make important educational decisions to guide instruction. Essential components of RtI in mathematics include the following:

- Three to four tiers of evidence-based instruction and intervention services
- A problem-solving method designed to inform the development of interventions
- An integrated data collection and assessment system to inform decisions at each tier (National Association of State Directors of Special Education 2005)

The RtI process relies on proactive, instructional problem solving among educators to develop dynamic instructional or intervention plans that are based on assessment data and that address academic or behavioral concerns about students. RtI in mathematics focuses on the effective use of evidence-based instructional approaches, resources, and strategies within the classroom while continuously monitoring student learning. Because the goal is to increase mathematical achievement for students, general education classroom teachers are crucial participants in the RtI process.

RtI models describe either a three- or four-tiered approach to providing services and interventions to students at increasing levels of intensity, as determined by student “response” through progress monitoring and data analyses of student results (Gersten et al. 2009). The rate of progress will guide important educational decisions, including eligibility for support services, such as special education (see fig. 1).

The tiers assure a continuous provision of instruction and interventions to address identified student needs and refer to the intensity of services that students may receive on the basis of assessment results relative to grade-level curriculum standards. These tiers use increasingly intense instructional methods and

Guiding questions for RtI instructional planning

1. What are the critical mathematical concepts and skills to be learned by all students?
2. How do the current resources in my classroom address the selected lesson’s mathematical concepts and skills of the standard I am to teach?
3. What prior mathematical knowledge do students need or have to master to reach the content standard?
4. What may be sources of difficulty and confusion for the students?
5. How can this lesson build on students’ prior mathematical knowledge and experiences?
6. What will students think and do in response to the instructional lesson?
7. What scaffolding and support can I provide to meet the needs of all learners through differentiating instruction and accommodating individuals?
8. Which questions, resources, strategies, activities, examples, and so on will clarify and/or extend conceptual learning by students?
9. Which grouping arrangements, accommodations, adaptations, use of levels of learning, cognitive/metacognitive strategies, and/or technology are needed for whole/small groups of students?
10. How can I make the mathematical learning task less complex without changing the goal?
11. What kinds of data are available and will help us assess students’ mathematics progress toward the set mathematical goals?
12. How will I check for mathematical conceptual understanding and depth of mathematical knowledge?
interventions. Data are collected continuously at each tier and are used to make instructional decisions to determine if students are responding to instruction and interventions.

**Planning to meet instructional needs**

With the diversity of students in today’s classrooms, assorted instructional practices are needed to engage everyone in learning and mastering mathematics. Instruction must address the strengths and needs of learners through the use of multiple resources and evidence-based instructional practices. As we consider the planning for RtI, we know that it is a model for effective practices for all learners and supports how we educate all learners. The philosophy of furnishing high-quality instruction based on individual student needs allows us to assess and evaluate student learning, how quickly a particular student acquires instructed material, and how to adjust our practices accordingly. By using assessment to determine a student’s learning rate and level of performance, we can make decisions about changes in instruction or educational goals for maximizing each student’s mathematical achievement. Teaching, therefore, calls for the knowledge and use of relevant instructional practices as well as multiple strategies and techniques that scaffold instruction to assure that students reach increasing levels of mathematical understanding (Berch and Mazzocco 2007; NCTM 2000).

RtI is a tiered service-delivery model, which means that different levels of service (instruction, assistance) are provided; students receive their instruction and any assistance at whatever tier (or level) they need that support. During the instructional process of planning, teaching, and assessing student learning, strategic use of evidence-based instructional practices, interventions, and various cognitive and metacognitive

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**Figure 1**

Progress Monitoring within Florida’s Problem-Solving and Response to Instruction/Intervention Framework

**Academic and Behavior Systems**

**Tier 3: Intensive, individualized interventions and supports**

The most intense (increased time, narrowed focus, reduced group size) instruction and intervention based on individual student need provided in addition to and aligned with tier 1 and tier 2 academic and behavioral instruction and supports

**Tier 2: Targeted supplemental interventions and supports**

More targeted instruction/intervention and supplemental support in addition to and aligned with academic and behavioral curriculum

**Tier 1: Core, universal instruction and supports**

General instruction and support provided to all students in all settings

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Strategies are important in meeting the needs of a wide range of students, particularly those who struggle with learning mathematics. Additionally, strategic use of instructional materials—manipulatives, graphic organizers specific to mathematics, visuals, and models—are necessary to support conceptual understanding of the mathematics and to give students a solid foundation for the continuous learning and mastery of math. As teachers plan and guide instruction and interventions on the basis of the needs of all students, questions may arise related to mastering mathematics standards (also see the sidebar on p. 22):

- Is the core curriculum effective? That is, do 80 percent of students meet the established benchmarks?
- Which students are at risk for failure?
- Does any over-representation of particular student groups appear in those students identified as at-risk or above grade level?

**A case study**

Let’s revisit Harwood’s class to describe the RtI in mathematics process as it is implemented within a classroom and school. Although each teacher receives state and school-wide assessment results at the beginning of the school year, Harwood administers various curriculum-based informal assessments related to grade-level curriculum benchmarks to determine her fourth graders’ mathematical strengths and weaknesses (see the online appendix for a sample of a fourth-grade math curriculum).

Moreover, Harwood carefully observes, takes notes, and talks to students to gain additional information about their knowledge in mathematics. Using the classroom data from assessments, observations, and discussions with students, Harwood identifies that place value, addition, and multiplication are skills that should be retaught, reinforced, and mastered so that several students (including Matthew) can be successful. Clearly, gaps in mathemati-
To further diagnose Matthew’s problems, Harwood draws on her conclusions and then takes specific steps to offer additional support on the basis of her findings. She knows that Matthew is able to memorize some of his facts. However, through teacher-student conferencing, she realizes that he has not fully developed or mastered the language for thinking about and describing multiplicative situations and their quantities, units, and equal groups. Harwood knows that Matthew’s lack of basic multiplicative understanding is preventing him from being able to solve the problem presented. Harwood continues her diagnoses by administering a curriculum-based informal assessment to further identify Matthew’s mathematical strengths and weaknesses (see fig. 2a). The scoring shows that he understands how to multiply a two-digit number by a one-digit number and that he shows mastery of the ones facts and the twos facts. Only one problem includes the threes facts, so analyzing his mastery of them cannot be determined with this curriculum-based measurement (CBM) probe. Harwood indicates that Matthew does not exhibit any significant error patterns when multiplying a two-digit number with a one-digit number. However, she does denote comments Matthew made about one specific problem that he corrected (see the Error Patterns comments in fig. 2b).

Harwood continues with the curriculum benchmarks, and the challenges to her student’s mathematical skills get harder. Matthew’s troubles become evident when she administers another CBM probe with multiplication that includes the sixes, sevens, eights, and nines facts (see fig. 3a). The scoring results show that Matthew has an understanding of some but not all of his multiplication facts. Harwood again analyzes the probe and writes Matthew’s strengths and weaknesses (see fig. 3b). Note the error patterns that occurred in the first problem when Matthew tried to solve $65 \times 8$. He does have mastery of his fives facts; he placed the 40 to show his answer. However, Matthew did not carry the 4 to the tens column above the 6 when he multiplied $5 \times 8$. Instead, he placed the 4 beside the 0 in the tens column of the answer section. He does not have mastery of the sixes facts; he attempted to solve $6 \times 8$ by writing tally marks. Although he wrote the correct number of tallies next to the problem, Matthew made an $x$ with his pencil near the top of the tallies and crossed out the problem without completing it. When Harwood later asked Matthew why he had not continued to work the problem, Matthew said the second problem looked easier and he wanted to move on.

The third problem in the first row ($61 \times 9$) reveals another common error pattern. Matthew multiplies $1 \times 9$ correctly and shows

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**Implementing RtI**

To effectively implement Response to Intervention in mathematics within our schools, multidisciplinary teams of educators must have both an awareness of and a commitment to developing and implementing instructional practices within the RtI process (Fletcher, Denton, and Tilly 2006). Below are some suggestions to consider.

- Read and be aware of current (and revisions to) policies and practices related to RtI and curriculum reforms in mathematics from the state department of education and the school district.
- Collaborate with colleagues about mathematics instruction, interventions, assessment practices, classroom accommodations, accountability, and current prereferral policies, procedures, and practices.
- Participate in school-wide educational teams (e.g., School Improvement Team, grade-level teams in mathematics, data teams, etc.) to learn about accountability policies and practices for all students, while sharing specific information related to students with and without disabilities who struggle in mathematics.
- Collect multiple sources of mathematics assessment data (e.g., formative, norm-referenced, observations, interviews, etc.) in a timely fashion to assure proactive problem solving and instructional planning for students experiencing difficulty.
- Research and provide professional development and products that are research-based to address identified instructional and behavioral student concerns in mathematics.
- Disseminate and model methods of differentiating instruction, use of instructional technology, and accommodations to be used in a general education classroom for the group or individual students to address documented mathematical needs.
- Administer norm-referenced and curriculum-based mathemathic assessments reliably and validly.
- Learn about and facilitate mathematical problem-solving methods on educational teams in school.
- Continue to read research and publications, attend webinars and professional development, and collaborate with colleagues to maintain current information as related to the implementation of Response to Intervention (RtI) in mathematics (see the online appendix).
knowledge of multiplication when zero and one are involved. However, instead of writing tallies as he did in the first problem to solve $6 \times 8$, he adds $6 + 9$ and places his answer of 15 in the appropriate column. He does not demonstrate strong knowledge of regrouping, adding regrouped numbers, or where regrouped numbers should be placed within a problem. The second row, third problem, is more evidence that he lacks this skill as he attempts to solve $32 \times 5$. Matthew multiplies $2 \times 5$ and places the 10 in the answer, but he does not place the number 1 above the multiplier so that he can add it when he multiplies numbers in the tens column. He tries to correct this mistake in another problem (row 3, fourth problem) when he multiplies $65 \times 9$. He seems to have some knowledge of place value and regrouping when he corrects his placement of the 4 above the 5 in the ones column, crossing out the 4 and placing it in the appropriate column. He also indicates knowledge of his fives facts, but he does not complete the problem. In the last problem, Matthew once again uses tally marks in his attempt at a solution. His mastery of the fives facts is evident, and he knows to place the answer in the tens column in the appropriate place-value holder, but he does not complete the answer.

(a) Matthew told his teacher that he got frustrated with writing so many tallies in the first problem and just gave up trying to solve it.

(b) The challenges to a student’s mathematical skills get harder with each assessment. Matthew’s troubles became evident after Harwood listed Matthew’s strengths and weaknesses and analyzed the second CBM probe that she administered.

1. Add and subtract numbers to 10,000.
2. Identify written and spoken numbers up to 100,000.
3. Explain in words how a math problem was solved.
4. Multidigit addition with regrouping
5. Multidigit subtraction with regrouping
6. Multiplication facts, factors to 9
7. Multiply 2-digit numbers by a 1-digit number.
8. Multiply 2-digit numbers by a 2-digit number.
9. Fluency with the multiplication combinations to $12 \times 12$
10. Division facts, divisors to 9
11. Divide 2-digit numbers by a 1-digit number.
12. Divide 3-digit numbers by a 1-digit number.
13. Add and subtract simple fractions with like denominators.
14. Add and subtract simple fractions with unlike denominators.
15. Identify fractional parts of an area.
16. Identify fractional parts of a group (of objects, people, etc.).
17. Read, write, and interpret fraction notation.
18. Order fractions with like and unlike denominators.
19. Read, write, and interpret decimal fractions in tenths and hundredths.
20. Add and subtract whole and mixed numbers.
22. Analyze and graph data.
23. Tell time to the nearest minute.
24. Relate number problems to everyday situations.
25. Use arrays, pictures or models of groups, and story contexts to represent multiplication situations.
The third CBM probe (see fig. 4) shows Matthew’s lack of knowledge with place value in multiplication. Although he showed mastery of multiplication of the easier facts, he did not exhibit evidence of adding with regrouping within the answers once he had completed the multiplication process, and thus his answers are incorrect. Further, he did not demonstrate knowledge of the place-value holder in the ones column as required when multiplying two-digit numbers by two-digit numbers. Students commonly misunderstand place value. Errors in Matthew’s work seem to indicate that perhaps place value could have been learned in isolation from previous knowledge and with little meaning (Baroody 1990). He lacks understanding of place-value structure when working with multiplication and does not seem to make the connection that zero will hold the place value, because he has ignored the 0 as the placeholder in each of the problems.

Through teacher-student conferencing and CBM probes, Harwood identified Matthew’s problem areas. She was able to use this analysis to discuss Matthew’s progress and any need for intervention. Harwood met with the RtI support team (an instructional coach, two grade-level teachers, the school psychologist, and a mathematics interventionist) to discuss her concerns and findings. Harwood described the instructional processes and strategies that she had been using in her classroom to teach mathematics during the first six weeks of school (see the online appendix for an example of a lesson plan). A plan of action for RtI was discussed and developed to help those students in need. Harwood would continue teaching the identified students during the general mathematics instructional time, and an interventionist would work with them for 20–30 minutes at a time, 3–5 days per week. This plan ensured that although the students received mathematics instruction during the regularly scheduled math time (tier 1), additional intensive instruction and support would be offered as tier 2 services. Intensive instruction did not focus on rote memorization or drill and practice but included additional authentic, concrete, hands-on, research-based activities and strategies to help students develop conceptual understanding of the mathematics concepts of place value, addition, and multiplication.

Matthew was grouped with three other students who were experiencing some of the same mathematical difficulties. During tier 2 activities, Matthew received explicit, systematic instruction with more opportunities to respond and verbalize his thought processes as well as view models of proficient problem solving. He was engaged in “doing” mathematics through hands-on experiences that related to real-life situations and incorporated multiple representations to increase his interest in math. Throughout Matthew’s work on the problems, he was asked to explain his thinking and justify his work using verbal think-alouds in addition to journaling and writing activities that helped him gauge his misunderstanding—and gaps in understanding—of the mathematical content presented to him. Opportunities were extended for Matthew to “think about” the material that
The progress-monitoring data showed significant positive impact on most students’ mastery of the math goals, but the RtI team developed an individual, tier 3 intervention plan to address Matthew’s specific instructional needs.

**FIGURE 5** Instructional Graph of Matthew’s Progress

![Graph showing Matthew's progress](image)

had been presented, and both Harwood and the interventionist communicated positive expectations and involved him in goal setting. They also used progress charts to give him immediate feedback on how he was doing.

The interventionist used warm-up activities, pretaught the mathematical vocabulary, and provided opportunities to review content that Matthew had not previously mastered. She presented content using direct and explicit methods to help make connections among the mathematics ideas, modeled what was expected through the appropriate mathematical representations, scaffolded approaches using graphic organizers, and used manipulatives during instruction. (For more intervention strategies, see the online appendix). Additionally, more time was spent on difficult tasks, fewer transitions took place, and mastery of content was determined before moving on to other mathematical tasks. To determine mastery and understanding of content, progress monitoring occurred biweekly with specific skills identified on each probe. Brief daily assessments (five problems) supplied continuous progress monitoring of student learning, with individual students graphing the results. Corrective feedback of student accuracy was offered immediately, which proved to be a powerful motivational tool to keep Matthew engaged in the word problems, particularly those he expressed were difficult for him to understand. Matthew responded well to verbal praise for his efforts and was eager to work through and become engaged in the problems to their completion.

As determined in the initial action plan, Harwood met with the RtI team after one month to discuss student progress, analyze data, and determine necessary revisions to the RtI action plan (see the online appendix for the intervention form). Judging from the progress-monitoring data, tier 1 instruction and the interventions during tier 2 had a significant positive impact on most students’ mastery of the identified math goals. However, progress-monitoring data also revealed that Matthew had not yet mastered the concept or procedures of multiplication (see fig. 5). To address this specific instructional need, the RtI team developed an individual, tier 3 intervention plan for Matthew, while he continued the current instructional and intervention plans from tiers 1 and 2. Instruction, interventions, and progress monitoring through the identified assessments would continue because of the positive student results on the identified grade-level curricular goals.

**Final thoughts**

Emphasis on accountability for high-quality instruction and interventions for all students has increased in math classrooms across the United States. Curriculum standards and expectations for student learning in math are changing. Therefore, our thinking about how we teach and how students learn math in our classrooms must also change. Teachers benefit from the continual feedback and school supports that help meet the needs of struggling students. Indeed, entire classrooms benefit from the concentration of resources that the RtI approach offers as the process focuses on students’ mathematical progress through the effective use of evidence-based instruction, interventions, and continuous progress monitoring in classrooms (IDEA 2004).

A model for effective practices for all learners, RtI supports how we educate all learners. The philosophy of providing high-quality instruction based on individual student needs allows us to assess and evaluate student learning as well as how quickly those students acquire instructed material. We can then adjust our practices accordingly. By using assessment to determine a student’s learning rate and level of performance,
we can make decisions about changes in instruction or educational goals for maximizing each student’s mathematical achievement. Teaching, therefore, calls for the knowledge and use of relevant instructional practices, as well as multiple strategies and techniques to scaffold instruction to ensure students reach increasing levels of mathematical understanding (Berch and Mazzocco 2007; NCTM 2000).

The benefits of implementing RtI in mathematics are plentiful: providing support in the general education setting, identifying students who are not succeeding within the general education curriculum, and offering early interventions to struggling students as well as to those who need a challenge (Fuchs and Deshler 2007; Riccomini and Witzel 2010). More benefits of shared responsibility for students include focusing on student data relevant to instruction and standards, promoting collaboration, and reducing the number of students referred for special education.

As teachers, we want to minimize the “wait-to-fail” approach and instead improve math learning through high-quality instruction and interventions within our classrooms, which is one of the greatest benefits of RtI (Fuchs and Fuchs 2005). Together, we can make a difference.

**BIBLIOGRAPHY**
Fuchs, Douglas, and Donald Deshler. 2007. “What We Need to Know about Responsiveness to Intervention and Shouldn’t Be Afraid to Ask.” *Learning Disabilities Research and Practice* 22 (2): 129–36.