Final Exam  Philosophy 60

Part 1: Multiple Choice, 1/2 pt. each.

Multiple Choice: (1 pt. each.)

1. \((p \rightarrow \neg p)\) is
   a. a tautology.
   b. a contradiction.
   c. contingent.
   d. a valid argument.

2. \(\neg(p \rightarrow q)\) is logically equivalent to
   a. \(\neg (\neg p v q)\)
   b. \((\neg p v \neg q)\)
   c. \((\neg q v \neg p)\)
   d. \((\neg p \rightarrow q)\)

3. \(p & \neg p\) is logically equivalent to
   a. \((p v p)\)
   b. \((p \leftrightarrow \neg p)\)
   c. \(\neg(p & \neg p)\)
   d. all of the above

4. “Serena is going to win unless Venus plays her best” is best represented in propositional logic as:
   (Let \(s\) stand for “Serena is going to win” and \(v\) stand for “Venus plays her best”)
   a. \((s \rightarrow \neg v)\)
   b. \((v \rightarrow s)\)
   c. \((\neg s \rightarrow v)\)
   d. \((\neg v \rightarrow \neg s)\)

5. An argument is deductively invalid if and only if
   a. one of the premises is false.
   b. all of its premises are false.
   c. the premises together with the denial of the conclusion forms an unsatisfiable set of sentences.
   d. the premises together with the conclusion forms a satisfiable set of sentences.

6. In the proof method known as Conditional Proof we do all of the following except
   a. assume the conditional is true.
   b. assume the antecedent of the conclusion.
   c. derive the consequent of the conclusion.
   d. prove a conditional conclusion.

7. Which of the following is not a formula of Q?
   a. \(a = b\)
   b. \(Fa \rightarrow p\)
   c. \(\forall x Fxy\)
   d. \(\exists x Fxa\)

8. Which of the following is a formula of Q?
   a. \(\forall x \exists y (Fxy \rightarrow \forall z Fxz)\)
   b. \(\exists x Fxx \lor \exists y Faa\)
   c. \(Fa=b\)
   d. \(\exists x Fa\)

9. “No boy chases all girls” is best expressed in the predicate calculus by:
   a. \(\neg \forall x \forall y ((Gx & By) \rightarrow Cxy)\)
   b. \(\neg \exists x \forall y (\neg (Gx & By) & Cxy)\)
   c. \(\neg \forall x \forall y ((Bx & Gy) & \neg Cxy)\)
   d. \(\neg \exists x \forall y ((Bx & \neg (Gy \rightarrow Cxy))\)

10. “All girls chase some shy boys” is best expressed in the predicate calculus by:
    a. \(\exists x Gx & Cxs\)
    b. \(\forall x \exists x ((Gx & By) \rightarrow Cxy)\)
    c. \(\forall x \exists y (Gx & (By \rightarrow (Sy \leftrightarrow Cxy)))\)
    d. \(\forall x \exists y ((Gx & By) & (Sy & Cxy))\)

11. Which of the following represents the proper use of the Quantifier Negation Rule? (Recall that \(\neg \neg \mid \neg\) is a different way of expressing logical equivalence.)
    a. \(\neg \forall x \neg (Fx v Gx) \mid \neg \forall x (\neg Fx & \neg Gx)\)
    b. \(\neg \forall x \neg (Fx v Gx) \mid \exists x \neg (\neg Fx & \neg Gx)\)
    c. \(\neg \forall x \neg (Fx v Gx) \mid \exists x (Fx v Gx)\)
    d. \(\neg \forall x \neg (Fx v Gx) \mid \neg \exists x Fx\)

12. “Frank is the only one who loves Jess” is best expressed by:
    a. \(Lfj & (\forall x (Lxj \rightarrow x=f))\)
    b. \(\exists x Lfj \& x=f\)
    c. \(\exists x (Lfx \& x=j)\)
    d. \(Lfj \& \forall x (Lfx \rightarrow x=j)\)

13. Which of the following pairs of formulas is explicitly contradictory?
    a. \(\exists x \forall y Fxy ; \neg \exists y \forall x Fxy\)
    b. \(\forall y \forall x Fxy ; \forall y \forall x \neg Fxy\)
    c. \(\exists x \forall y Fxy ; \neg \exists y \forall x Fxy\)
    d. \(\neg \exists x \forall y Fxy ; \exists x \forall y Fxy\)

14. Which of the following inferences can definitely not be achieved in a single step by natural deduction in Q?
    a. \(\forall x Fx \mid \neg \neg \forall x Fx\)
    b. \(\forall x \neg Fx \mid \neg \exists x Fa\)
    c. \(\exists x \neg Fx \mid \neg Fa\)
    d. \(\neg Fa \mid \neg \exists x Fx\)

15. Which of the following is the best translation of: “Amie’s only disciple is Bill’s father’’?
    a. \(\exists y (y = d(a) \& y = f(b))\)
    b. \(\exists x Dxa \& \forall y (Dya \rightarrow y = f(b))\)
    c. \(\exists y (Dya = (y \leftrightarrow f(m)))\)
    d. \(\forall x (Fxb \rightarrow Dxa)\)
16. The conjunction of two contingent sentences is itself always
   a. contingent.
   b. contradictory.
   c. valid.
   d. none of the above.

17. When all branches of a truth tree performed on the formula
    \( \neg A \) close, then:
   a. \( A \) is valid.
   b. \( A \) is a contradiction.
   c. \( \neg A \) is satisfiable.
   d. \( \neg A \) is contingent.

18. A temporary dispatch mark (*) beside a formula in a truth
    tree means:
   a. the formula may not be reapplied later.
   b. the formula is governed by an unrestricted rule.
   c. the truth tree is non branching.
   d. none of the above.

19. In the truth table for the following formula, the column under
    the main connective would contain
    \( ((p \lor p) \leftrightarrow (p \land p)) \)
   a. all T's.
   b. all F's.
   c. T's and F's.
   d. either T's or F's, but not both T's and F's.

20. Line 3 in the following proof:

   1. \( Fg(a,b) \) \hspace{1cm} A
   2. Show \( \exists x Fx \) \hspace{1cm} A
   3. \( \exists x Fx \) \hspace{1cm} \( \exists I, 1 \)
   a. is correct.
   b. is incorrect because you can’t do \( \exists I \) on two different
      constants.
   c. is incorrect because it skips a step.
   d. is incorrect because it violates a restriction on \( \exists I \).
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Proofs (3 pts. each)

1. Use a truth tree to determine whether the following sentence is valid.

   \(((p \rightarrow (p \rightarrow q)) \rightarrow (\neg(p \& \neg q) \rightarrow \neg p)) \) 

2. Use natural deduction to prove the following:

   \(((p \leftrightarrow q) \& (\neg p \leftrightarrow r)) \therefore \neg (q \& r)\)

3. Use a truth tree to determine whether the following argument is valid.

   \((\exists x Fx \& \neg \exists y Fy) \therefore \forall z (Fz \rightarrow \neg Fz)\)

4. Prove the following using natural deduction.

   \((\forall x \exists y Fxy \lor \exists x \forall y Fyx) \therefore (\neg \exists y Fcy \rightarrow \exists z Fdz)\)

5. Prove using natural deduction.

   \(\forall x \forall y f(g(y)) = g(f(x)); \exists x \exists y f(x) = g(y) \therefore \exists x \exists y f(f(x)) = g(g(y))\)