Finance Practice Problems

Interest

Interest is the cost of borrowing money. An interest rate is the cost stated as a percent of the amount borrowed per period of time, usually one year. The prevailing market rate is composed of:

1. The Real Rate of Interest that compensates lenders for postponing their own spending during the term of the loan.
2. An Inflation Premium to offset the possibility that inflation may erode the value of the money during the term of the loan. A unit of money (dollar, peso, etc) will purchase progressively fewer goods and services during a period of inflation, so the lender must increase the interest rate to compensate for that loss.
3. Various Risk Premiums to compensate the lender for risky loans such as those that are unsecured, made to borrowers with questionable credit ratings, or illiquid loans that the lender may not be able to readily resell.

The first two components of the interest rate listed above, the real rate of interest and an inflation premium, collectively are referred to as the nominal risk-free rate. In the USA, the nominal risk-free rate can be approximated by the rate of US Treasury bills since they are generally considered to have a very small risk.

Simple Interest

Simple interest is calculated on the original principal only. Accumulated interest from prior periods is not used in calculations for the following periods. Simple interest is normally used for a single period of less than a year, such as 30 or 60 days.

\[ F = P \times (1 + rt) \]
\[ I = A - P \]
\[ I = P \times rt \]

F: Future Value of the investment
P: principal (initial value of an investment)
r: annual interest rate in percentage (%)
I: interest after t years.

Example 1: Michelle invested $5000.00 in mutual fund with the interest rate of 4.8%. How much interest would she earn after 2 years?

Answer:

\[ P = 5000.00 \quad r = 4.8\% \quad t = 2 \]
\[ I = P \times rt \]
\[ I = (5000.00)(4.8\%)(2) = 480.00 \]

Hence, Michelle would earn $480 after 2 years.

Example 2.

Jeff has one savings account with the interest rate of 3.3%, and one money market account with the interest rate of 5.1% in a bank. If he deposits $1200.00 to the savings account, and $1800.00 to the money market account, how much money will he have after 6 years?

Answer:

Savings account:
\[ P = $1200.00 \quad r = 3.3\% \quad t = 6 \]

\[ F = P(1 + rt) \]
\[ F = ($1200.00)[1 + (3.3\%)(6)] = ($1200.00)(1.198) = $1437.60 \]

Money market account:

\[ P = $1800.00 \quad r = 5.1\% \quad t = 6 \]

\[ F = P(1 + rt) \]
\[ F = ($1800.00)[1+(5.1\%)(6)] = ($1800.00)(1.306) = $2350.80 \]

Total amount: \$1437.60 + $2350.80 = $3788.40

Hence, Jeff will have \$3788.40 after 6 years.

\[
\text{Compound Interest} \quad F = P \left(1 + \frac{r}{n}\right)^{nt}
\]

Compound interest is calculated each period on the original principal and all interest accumulated during past periods. Although the interest may be stated as a yearly rate, the compounding periods can be yearly, semiannually, quarterly, or even continuously.

You can think of compound interest as a series of back-to-back simple interest contracts. The interest earned in each period is added to the principal of the previous period to become the principal for the next period. For example, you borrow \$10,000 for three years at 5% annual interest compounded annually:

interest year 1 = \( p \times i \times n = 10,000 \times .05 \times 1 = 500 \)
interest year 2 = \( (p_2 = p_1 + i_1) \times i \times n = (10,000 + 500) \times .05 \times 1 = 525 \)
interest year 3 = \( (p_3 = p_2 + i_2) \times i \times n = (10,500 + 525) \times .05 \times 1 = 551.25 \)

Total interest earned over the three years = \( 500 + 525 + 551.25 = 1,576.25 \). Compare this to 1,500 earned over the same number of years using simple interest.

The power of compounding can have an astonishing effect on the accumulation of wealth. This table shows the results of making a one-time investment of \$10,000 for 30 years using 12% simple interest, and 12% interest compounded yearly and quarterly.

<table>
<thead>
<tr>
<th>Type of Interest</th>
<th>Principal Plus Interest Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>46,000.00</td>
</tr>
<tr>
<td>Compounded Yearly</td>
<td>299,599.22</td>
</tr>
<tr>
<td>Compounded Quarterly</td>
<td>347,109.87</td>
</tr>
</tbody>
</table>
Example 1:

Alex has $4864.77 in his money market account currently. If the annual interest rate os 3.9%, and the interest is compounded biweekly, how much money was in his account 2 years ago? (1 year = 52 weeks)

Answer:

\[ F = 4864.77 \quad r = 3.9\% \quad n = 26 \quad t = 2 \]

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ 4864.77 = P \left[1 + \left(\frac{3.9\%}{26}\right)\right]^{26(2)} = P\left[1 + 0.0015\right]^{52} \]

\[ P = \frac{4864.77}{1.0015^{52}} \approx 4500.00 \]

Hence, $4500.00 were in Alex's account 2 years ago.

Example:

Emily opens a savings account in a bank with the annual interest rate of 2.7%. If she deposits $6000.00 to the account, and the interest is compounded continually, how much interest will she earn after 4 years?

Answer:

\[ P = 6000.00 \quad r = 2.7\% \quad t = 4 \]

\[ I = P \times (e^t - 1) \]

\[ I = (6000.00)(e^{0.027}(4) - 1) = (6000.00)(e^{0.108} - 1) \approx 684.29 \]

Hence, Emily will earn $684.29 after 4 years.
Ordinary Annuity (Sinking Fund)

Payment at the end of each period

\[ F = R \left( \frac{1 + \frac{r}{n}}{\frac{r}{n}} \right)^{nt} - 1 \]

Example: Joe deposits $22,000 at the end of each year for 7 years, in an account paying 6% compounded annually, how much will he have on deposit after 7 years? Ans: $184,664.43

Practice 1: Mina deposits $500 at the end of each month for 10 years, in an account paying 5% compounded monthly, how much will she have on deposit after 10 years?

Practice 2: Napoleon deposits $1,200 at the end of each quarter for 10 years, in an account paying 8% compounded quarterly, how much will he have on deposit after 10 years? Ans: $72,482.38

Practice 3-a: Jose wants to retire in twenty years and for this purpose he is depositing $200 at the end of each month in a sinking fund that pays 7.2% compounded monthly. If he will be doing this for twenty years, then how much money will be there for him when he retires? Ans: $106,752.47

Practice 3-b: If Joe wants to accumulate $130,000 in the twenty years period, and then what interest rate would provide that amount? Ans: 8.79%

Practice 4: Find the amount of payment to be Joe needs to make into a sinking fund every quarter to accumulate $62,000 after 6 years: Knowing that money earns 8% compounded quarterly. Ans: $2,038.01

Practice 5: Find the amount of payment to be made into a sinking fund to accumulate $75,000 for 4 and half year: money earns 6% compounded semiannually. Ans: $7,382.54

Annuity Due

Payment at the beginning of each period

\[ F = R \left( \frac{1 + \frac{r}{n}}{\frac{r}{n}} \right)^{nt+1} - 1 \]

Example: Joe deposits $500 at the beginning of each quarter end for 7 years, in an account paying 12% compounded quarterly, how much will he have on deposit after 7 years? Ans: $22,109.43
Example: Joe deposits $500 at the end of each quarter for 7 years, in an account paying 12% compounded quarterly, how much will he have on deposit after 7 years?  
\text{Ans:} \$21,465.46

Practice 1: Cesar deposits $16,000 at the beginning of each year for 8 years, in an account paying 4.7% compounded annually, how much will he have on deposit after 8 years?  
\text{Ans:} \$158,260.36

Practice 2: Cesar deposits $100 at the beginning of each quarter for 30 years, in an account paying 4% compounded annually, how much will he have on deposit after 8 years?  
\text{Ans:}

Practice 3: Find the amount of payment to be Joe needs to make into an annuity fund every quarter to accumulate $62,000 after 6 years: Knowing that money earns 8% compounded quarterly.  
\text{Ans:}

Practice 4: Find the amount of payment to be made into an annuity fund to accumulate $75,000 for 4 and half year: money earns 6% compounded semiannually.  
\text{Ans:}

Amortization

\[
R = P \left[ \frac{r}{n} \right] \frac{n t}{1 - (1 + \frac{r}{n})^{-n t}}
\]

\[
P = \text{Loan Amount} \quad R = \text{Periodic Payment} \quad r = \text{Interest Rate}
\]

\[
n = \text{Compounding Period} \quad t = \text{Time}
\]

Example (4-year payment): A car costs $22,000. After a down payment of $4,000, the balance will be paid off in 48 equal monthly payments with the interest of 12% per year on the unpaid balance. Find the amount of each payment.  
\text{Ans:} \$474.01

Practice 1 (5-year term): A car costs $22,000. After a down payment of $4,000, the balance will be paid off in 60 equal monthly payments with the interest of 12% per year on the unpaid balance. Find the amount of each payment. \text{Ans:}

Practice 2 (6-year term): A car costs $22,000. After a down payment of $4,000, the balance will be paid off in 72 equal monthly payments with the interest of 12% per year on the unpaid balance. Find the amount of each payment. \text{Ans:}
Practice 3 (Bad credit): A car costs $22,000. After a down payment of $4,000, the balance will be paid off in 48 equal monthly payments with the interest of 18% per year on the unpaid balance. Find the amount of each payment. \textit{Ans:} \[ \] 

Practice 4 (Good Credit and 4-year term): A car costs $22,000. After a down payment of $4,000, the balance will be paid off in 48 equal monthly payments with the interest of 6% per year on the unpaid balance. Find the amount of each payment. \textit{Ans:} \[ \] 

Practice 5 (Bad Credit and 5-year term): A car costs $22,000. After a down payment of $4,000, the balance will be paid off in 60 equal monthly payments with the interest of 18% per year on the unpaid balance. Find the amount of each payment. \textit{Ans:} \[ \] 

Practice 6 (Bad Credit and 7-year term): A car costs $22,000. After a down payment of $4,000, the balance will be paid off in 72 equal monthly payments with the interest of 18% per year on the unpaid balance. Find the amount of each payment. \textit{Ans:} \[ \] 

Additional Problems

1) How many days will it take for a sum of $1,500 to earn $25 interest if it is deposited in a bank paying 5% a year? (Use a 365-day a year.) \textit{Ans:} 121.67 days

2) How long will it take an investment of $5,000 to triple if the investment earns interest at the rate of 8% a year compounded quarterly? \textit{Ans:} 13.87 years

3) Today, the price of a gallon of milk is $4.25, assuming inflation rate to be 4.5% a year. What will be the price of a gallon of milk 10 years from now? \textit{Ans:} $6.67

4) Five and a half years ago, Chris invested $10,000 in a retirement fund that grew at the rate of 10.82% per year compounded quarterly. What is his account worth today? \textit{Ans:} $17,989.33
5) 5 years ago, Johnny Cash invested a sum of money in a saving account with interest of 8% per year compounded quarterly. His investment is now worth $22,289.22. How much did he originally invest? *Ans: $15,000*

6) Find the future value of ordinary annuity for $150 per month for 15 years at 10% per year compounded monthly?  
*Ans: $62,170.55*

7) Find the present value of ordinary annuity for $150 a month at 8% per year compounded quarterly for 10 years?  
*Ans: $4,103.32*

8) If you contribute $5,000 a year into a trust account, then how much will be in the account after 25 years if the account earns interest at the rate of 8.5% per year compounded yearly? *Ans: $38,433.81*

9) Pope invested only $24,000 in a retirement fund 5 years ago. Today his investment is worth $34616. Find the effective annual rate of return on his investment over 5-yr period. *Ans:*

10) Find the rate of interest per year compounded on a daily basis that is equivalent to 9.6% per year compounded monthly. *Ans:*

11) If $54,000 is invested at an interest rate of 9% for 7.5 years compounded continuously, then find its future value.  
*Ans: $106,057.78*

12) Find the monthly house payments for a loan of 188,000 at 5.74% for 15 years *Ans: $2,122.91*

13) Find the future value of an annuity of $672 deposited at the beginning of each quarter for 7 years at 8% compounded quarterly.  
*Ans: $24,898.41*
14) A company has ordered 20 new PCs at a cost of $1800 each. They will not be delivered for 5 months. What amount should the firm deposit in an account paying 8.1% to have enough money to pay for them? \( \text{Ans: } $34,824.67 \)

15) A pack-a-day smoker spends about $120 per month on cigarettes. Suppose the smoker invests that amount at the end of each month in a savings account at 6.7% compounded monthly. What would the account be worth after 45 years? \( \text{Ans: } $413,061.41 \)

16) The Blues Clues family bought a house for $315,000. They paid $20,000 down and took out a 30-year mortgage for the balance at 7%. Find their monthly rent. \( \text{Ans: } $1,962.64 \)

17) Find the total interest Blues Clues family will pay. \( \text{Ans: } $391,550.4 \)

18) Find the amount of each payment that must be paid into a sinking fund to accumulate $6,000 at 8% compounded monthly for 3 years. \( \text{Ans: } $148.02 \)

19) If money can be borrowed at 8% compound monthly, which one is larger: $10,000 now or $15,000 in 5 years? Use present value to decide. \( \text{Ans: }$15,000 in 5 years \)

20) One of us classmates needs to borrow $18,000 for 1 year. He has been offered a loan with interest compounded monthly and a compound amount of $19,952.42. Find the rate. \( \text{Ans: } 10.34\% \)

21) Billy Jean King deposited $6500 in an account paying 7.5% compounded quarterly. After 3 years the rate drops to 4% compounded semiannually. Find the amount in her account at the end of 7 years. \( \text{Ans: } $9,517.58 \)

22) For one year, a student loan of $52,000 at 9% compounded semiannually resulted in a maturity value of $5,934.06. \( \text{Ans: } 1.96 \text{ year, 1yr,11months} \)
23) Bobby Cash deposited $10,000 at 8% compounded quarterly. Two years after she makes the first deposit, he adds another $20,000, also at 8% rate compounded quarterly. What total amount will he have 4 years after his first deposit? \textit{Ans:}\ $37,161.04$

24) Bobby Cash deposited $10,000 at 8% compounded quarterly. Two years after she makes the first deposit, he adds another $20,000, also at 8% rate compounded quarterly. What total amount will he have 6 years after his first deposit? \textit{Ans:}\ $43540.08$

25) John and Jill have $20,000 cash for the down payment of a house and they can afford a 15-year mortgage payment of $2,500/month. If the best mortgage rate that they can get is 7.5% then what will be the most affordable home that they can buy by their current budget plan? \textit{Ans:}\ $269,683.58 + 20,000 = 289,683.57 = 290,000$

26) Adam and Eve need to borrow $115,000 to purchase a cave and are debating whether they should use a 20-year mortgage or 30-year mortgage. They also want to know the effect of two interest rates, a 6% and 8%, on

\begin{itemize}
  \item[\textbf{a)}] Monthly payment
  \item[\textbf{b)}] Total cost
  \item[\textbf{c)}] Total interest paid
\end{itemize}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Term of the mortgage} & \textbf{6\%} & \textbf{8\%} \\
\hline
20 years & 823.89 & 961.40 \\
30 years & 690.00 & 844.10 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
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\begin{tabular}{|c|c|c|}
\hline
\textbf{Term of the mortgage} & \textbf{6\%} & \textbf{8\%} \\
\hline
20 years & 197,616.60 & 232,736 \\
30 years & 248,400 & 303,876 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Term of the mortgage} & \textbf{6\%} & \textbf{8\%} \\
\hline
20 years & 82616 & 117736 \\
30 years & 133400 & 188876 \\
\hline
\end{tabular}
\end{table}