Abe Mirza
Finance

\[ F = P(1 + \frac{r}{n})^{nt} \]

- \( F \) = Future Value
- \( P \) = Present Value
- \( t \) = time in years
- \( r \) = rate
- \( n \) = Compounding periods (how often the interest is added to the principle)

Ex 1. If $4000.00 is invested at an interest rate of 6% for 10 years at different compounding periods, then find the future value and total interest for each case.

<table>
<thead>
<tr>
<th>Compounding Period</th>
<th>Future Value</th>
<th>Total Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually, ( n = 1 )</td>
<td>( F = 4000(1 + \frac{0.06}{1})^{10} = 7163.39 )</td>
<td>( I = 7163.39 - 4000 = 3163.39 )</td>
</tr>
<tr>
<td>( r = 7% )</td>
<td>( F = 4000(1 + \frac{0.07}{1})^{10} = 7868.61 )</td>
<td>( I = 7868.61 - 4000 = 3868.61 )</td>
</tr>
<tr>
<td>( r = 8% )</td>
<td>( F = 4000(1 + \frac{0.08}{1})^{10} = 8635.70 )</td>
<td>( I = 8635.70 - 4000 = 4635.70 )</td>
</tr>
<tr>
<td>Semiannually, ( n = 2 )</td>
<td>( F = 4000(1 + \frac{0.06}{2})^{20} = 7224.44 )</td>
<td>( I = 7224.44 - 4000 = 3224.44 )</td>
</tr>
<tr>
<td>Quarterly, ( n = 4 )</td>
<td>( F = 4000(1 + \frac{0.06}{4})^{40} = 7256.07 )</td>
<td>( I = 7256.07 - 4000 = 3256.07 )</td>
</tr>
<tr>
<td>Monthly, ( n = 12 )</td>
<td>( F = 4000(1 + \frac{0.06}{12})^{120} = 7277.59 )</td>
<td>( I = 7277.59 - 4000 = 3277.59 )</td>
</tr>
<tr>
<td>Daily, ( n = 365 )</td>
<td>( F = 4000(1 + \frac{0.06}{365})^{3650} = 7288.12 )</td>
<td>( I = 7288.12 - 4000 = 3288.12 )</td>
</tr>
</tbody>
</table>
**Practice Problems**

\[ F = P\left(1 + \frac{r}{n}\right)^{nt} \]

- \( F \) = Future Value
- \( P \) = Present Value
- \( t \) = time in years
- \( r \) = rate
- \( n \) = Compounding periods (how often the interest is added to the principle)

A: If $1000.00 is invested at an interest rate of 9% for 15 years at different compounding periods, find the **future value** and effective interest rate for each case.

| Compounding Period  | Future Value | Total Interest  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( I = F - P )</td>
</tr>
<tr>
<td>Annually  ( n = 1 )</td>
<td>Answer:$3642.48</td>
<td>Answer:$2642.48</td>
</tr>
<tr>
<td>Semiannually  ( n = 2 )</td>
<td>Answer:$3745.32</td>
<td>Answer:$2745.32</td>
</tr>
<tr>
<td>Quarterly  ( n = 4 )</td>
<td>Answer:$3800.13</td>
<td>Answer:$2800.13</td>
</tr>
<tr>
<td>Monthly  ( n = 12 )</td>
<td>Answer:$3838.04</td>
<td>Answer:$2838.04</td>
</tr>
<tr>
<td>Daily  ( n = 365 )</td>
<td>Answer:$3856.78</td>
<td>Answer:$2856.78</td>
</tr>
</tbody>
</table>
Present Value

**Ex 2:** A company has agreed to pay $2.4 million in 5 years to settle a law suit. How much must they invest now in an account paying 8% with compounding periods to have that amount when it is due?

\[ F = P(1 + \frac{r}{n})^{nt} \]

<table>
<thead>
<tr>
<th>Compounding Period</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually ( n = 1 )</td>
<td>[ 2.4 = P(1 + \frac{0.08}{1})^{1(5)} ] [ 2.4 = P(1.08)^5 ] [ 2.4 = P(1.4693) ] [ P = \frac{2.4}{1.4693} = 1.6334 \text{ million} ]</td>
</tr>
<tr>
<td>Semiannually ( n = 2 )</td>
<td>[ \text{Ans :$1.6214$ million} ]</td>
</tr>
<tr>
<td>Quarterly ( n = 4 )</td>
<td>[ \text{Ans :$1.6151$ million} ]</td>
</tr>
<tr>
<td>Monthly ( n = 12 )</td>
<td>[ \text{Ans :$1.6109$ million} ]</td>
</tr>
<tr>
<td>Daily ( n = )</td>
<td>[ \text{Ans :$1.6088$ million} ]</td>
</tr>
</tbody>
</table>
A company has agreed to pay $4.5 million in 10 years to settle a lawsuit. How much must they invest now in an account paying 9% with compounding periods to have that amount when it is due?

<table>
<thead>
<tr>
<th>Compounding Period</th>
<th>Present Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually n =</td>
<td>Ans: $1.901 million</td>
</tr>
<tr>
<td>Semiannually n =</td>
<td>Ans: $1.8659 million</td>
</tr>
<tr>
<td>Quarterly n =</td>
<td>Ans: $1.8479 million</td>
</tr>
<tr>
<td>Monthly n =</td>
<td>Ans: $1.8356 million</td>
</tr>
<tr>
<td>Daily n =</td>
<td>Ans: $1.8298 million</td>
</tr>
</tbody>
</table>

Solving for t(time) and r(rate) by trials and error.

C: How long will $25,000 investment need to result in a future value of $40,000? The interest rate is 6% compounded monthly.

\[ 40,000 = 25,000 \left(1 + \frac{0.06}{12}\right)^{12t} \]

\[ 40,000 / 25000 = (1.005)^{12t} \]

\[ 1.6 = (1.005)^{12t} \]

Then try to pick different t values that make the equation \( 1.6 = (1.005)^{12t} \) work.

D: How long will $50,000 investment need to result in a future value of $88,000? The interest rate is 8% compounded semi-annually.

E: How long will $180,000 investment take to result in a future value of $230,000? The interest rate is 4.5% compounded monthly.

F: At what interest rate will $20,000 investment need to result in a future value of $45,000 if the money is kept in 20 years? Assuming it is compounded semi-annually.

G: At what interest rate will $25,000 investment need to result in a future value of $45,000 if the money is kept in 10 years? Assuming it is compounded monthly.
Name_____________________________________________________

1.) Your 3 year investment of $20,000 received 5.2% interested compounded semi annually. What is your total return?
Answer: $23,329.97

2.) You borrowed $59,000 for 2 years at 11% which was compounded annually. What total will you pay back?
Answer: $72,693.90

3.) Your allowance of $190 got 11% compounded monthly for 1 2/3 years. What’s it worth after the 1 2/3 years?
Answer: $228.04

4.) Your 6 1/4 year investment of $40,000 at 14% compounded quarterly is worth how much now?
Answer: $94,529.80

5.) You borrowed $1,690 for 5 1/2 years at 5.7% compounded semi annually. What total will you pay back?
Answer: $2,176.33

6.) Your $440 gets 5.8% compounded annually for 8 years. What will your $440 be worth in 8 years?
Answer: $690.78

7.) Your $54,200 2 year car loan is at 15.1% compounded annually. What will you have paid for your car after 2 years?
Answer: $71,804.21

8.) You invest $55 at 10% compounded annually for 3 years. How much will your investment be worth in 3 years?
Answer: $73.21

9.) Your 8 year loan of $12,200 is at 5.3% compounded annually. How much will you have paid in total for your loan?
Answer: $18,441.10

10.) You invest $1,900 at 4% and it’s compounded semi annually for 3 years. How much will your $1,900 be worth in 3 years?
Answer: $2,139.71

© http://math.about.com
Applications of compound interest

Solve each problem. Show all work.

1. How much money will you have in 6 years if you invest $5000 at 4 1/2 % compounded monthly?

   1)____________

2. What interest rate do you need for a $4000 investment to double in 12 years?

   2)____________

3. How much money do you need to invest at 4 % in order to have $10,000 after 8 years?

   3)____________

4. How much money will you have in 4 months if you invest $2000 at 4% compounded monthly?

   4)____________

5. How much interest will you earn in 10 years if you invest $8500 at 4 1/4 % compounded semi-annually?

   5)____________

6. In 1995, the population of Math Valley was 18,000. If the population is increasing at an annual rate of 2.5%, what was the population in 2015?

   6)____________
7. A certain species of bird is in danger of becoming extinct. There were 1600 birds in 2000 and they are decreasing at an annual rate of 5.6%.

a) If this trend continues, how many birds will be left by 2015? 7a) ____________

b) How many birds would there have been in 1995? 7b) ____________

8. How much money would you need to deposit today at 5% annual interest compounded quarterly to have $16,000 in the account after 9 years? 8) ____________

9. How much money do you need to invest at 3.2% compounded daily in order to have $15,500 at the end of 10 years? 9) ____________

10. If you deposit $2500 into an account paying 11% annual interest compounded quarterly, how long until there is $4500 in the account? 10) ____________