Matrix

Given $A=\begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$, $C=\begin{bmatrix} -1 & 2 \end{bmatrix}$ $D=\begin{bmatrix} 15 & 25 \\ 32 & 18 \\ 45 & 55 \end{bmatrix}$ $E=\begin{bmatrix} 38 & 62 \\ 22 & 28 \\ 15 & 35 \end{bmatrix}$ $F=\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

$G=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $H=\begin{bmatrix} -1 & 0 & 2 \end{bmatrix}$

Compute the followings; Please check this website!!!! It can check your work. **http://www.uni-bonn.de/~manfear/matrixcalc.php**

1. $A-B$ **2.** $B-2A$ **3.** $D-E$ **4.** $E-\frac{1}{5}D$

5. $P(D)$ **6.** $P(E)$ **7.** $P(D+E)$ **8.** $P\left(E+\frac{1}{5}D\right)$

9) Solve for X, when $2X=-2A-B$

10) Solve for X, when $5X=-3A+2B$

11) Solve for X, when $-X=A-3B$

12) Solve for X, when $-\frac{1}{2}X=2A-B$

Matrix Multiplications

Compute the followings wherever is applicable;

1. AB **2.** BA **3.** A^2 **4.** A^3

5. B^2 **6.** B^3 **7.** AB **8.** AC

9. CA **10.** BC **11.** CB **12.** DA

13. BE **14.** DG **15.** EH **16.** CG

17. GC **18.** FH **19.** HF **20.** AH

Finding the inverse of a matrix if it exists.

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \Rightarrow \quad A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \Rightarrow \quad A^{-1}A=AA^{-1}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$ad-bc$ is called the **determinant** of matrix A and if $ad-bc=0$ A^{-1} does not exists.

$$A=\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix} \quad \Rightarrow \quad A^{-1}=\frac{1}{3(2)-(-1)(-2)}\begin{bmatrix} 2 & -(-1) \\ -(-2) & 3 \end{bmatrix}=\frac{1}{4}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}=\frac{1}{4}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}=\begin{bmatrix} 2/4 & 1/4 \\ 2/4 & 3/4 \end{bmatrix}$$

$$A=\begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix} \quad \Rightarrow \quad A^{-1}=\frac{1}{3(2)-(-1)(-6)}\begin{bmatrix} 2 & -(-1) \\ -(-6) & 3 \end{bmatrix}=\frac{1}{0}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix} \quad \Rightarrow \quad A^{-1} \textbf{ does not exists}$$

Find the inverse of the following matrices:

1) $A=\begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix}$ **2)** $A=\begin{bmatrix} -4 & -6 \\ 2 & 3 \end{bmatrix}$ **3)** $A=\begin{bmatrix} 2/7 & -3/4 \\ 1/5 & 4/5 \end{bmatrix}$

4) $A=\begin{bmatrix} 7 & -12 \\ -8 & -5 \end{bmatrix}$ **5)** $A=\begin{bmatrix} -1/4 & 9/4 \\ 5/3 & 8/9 \end{bmatrix}$ **6)** $A=\begin{bmatrix} -12 & 3 \\ 5 & -2 \end{bmatrix}$

Solving a system of equations by $X=A^{-1}b$ method.

$$AX=b \qquad X=A^{-1}b \qquad \begin{cases} 4x-2y=10 \\ 3x-5y=11 \end{cases} \qquad A=\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} \quad X=\begin{bmatrix} x \\ y \end{bmatrix} \quad b=\begin{bmatrix} 10 \\ 11 \end{bmatrix}$$

$$A^{-1}=\frac{1}{4(-5)-(-2)(3)}\begin{bmatrix} -5 & 2 \\ -3 & 4 \end{bmatrix}=\frac{1}{-14}\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$$

$$X=\begin{bmatrix} x \\ y \end{bmatrix}=A^{-1}b=\frac{1}{-14}\begin{bmatrix} -5 & 2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 10 \\ 11 \end{bmatrix}=\frac{1}{-14}\begin{bmatrix} -28 \\ 14 \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \end{bmatrix}=\begin{bmatrix} x \\ y \end{bmatrix} \quad \Rightarrow \quad x=2, \quad y=-1 \quad =$$

Solving each system of equations by $X=A^{-1}b$ method.

1) $\begin{cases} x+2y=10 \\ 3x-5y=-3 \end{cases}$ **2)** $\begin{cases} 2x+9y=17 \\ -3x-2y=-27 \end{cases}$ **3)** $\begin{cases} 3x - y=12 \\ -6x+2y=-18 \end{cases}$ **4)** $\begin{cases} 3x-2y=3 \\ 2x+3y=-11 \end{cases}$

Markovian Process

- It is studied that by the end of the next year 30 % of SBC Customers will switch to other long distance carrier and 70 % remain SBC customers, on the other hand 40 % customers of long distance other carrier will switch to SBC and the rest will stay with their current carrier. If the trend continues for the next three years and at the present time there are 3 million SBC customers and 1 million from other carriers, then set up the **markov chain** matrix for this marketing study for the first three years A, A^2, A^3 and answer the following questions;

1. The probability that an SBC customer remain an SBC customer two years from now.

2. The probability that an SBC customer remain an SBC customer three years from now.

3. The probability that an SBC customer will switch to another carrier two years from now.

4. The probability that an SBC customer will switch to another carrier three years from now.

5. The probability that a **non-** SBC customer remain a non - SBC customer two years from now.

6. The probability that a **non-** SBC customer remain a **non -** SBC customer three years from now.

7. The probability that a **non-** SBC customer switch to SBC customer two years from now.

8. The probability that a **non-** SBC customer switch to SBC customer three years from now.

As it was stated at the present time there 3 million SBC customers and 1 million from other carriers, then answer the following questions.

9. How many SBC customers will be there two years from now?

10. How many SBC customers will be there three years from now?

11. How many non- SBC customers will be there two years from now?

12. How many non SBC customers will be there three years from now?

Matrix Multiplications

Please check this website!!!! It can check your work. **http://www.uni-bonn.de/~manfear/matrixcalc.php**

1. $AB = \begin{bmatrix} 8 & 3 \\ -2 & -1 \end{bmatrix}$ **2.** $BA = \begin{bmatrix} 2 & -3 \\ -4 & 5 \end{bmatrix}$ **3.** $A^2 = \begin{bmatrix} 4 & -9 \\ 0 & 1 \end{bmatrix}$ **4.** $A^3 = \begin{bmatrix} 8 & -21 \\ 0 & 1 \end{bmatrix}$

5. $B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **6.** $B^3 = \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$ **7.** $AG = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$ **8.** $AC = NA$

9. $CA = \begin{bmatrix} -2 & 5 \end{bmatrix}$ **10.** $BC = NA$ **11.** $CB = \begin{bmatrix} -5 & -2 \end{bmatrix}$ **12.** $DA = \begin{bmatrix} 30 & -20 \\ 64 & -78 \\ 90 & -80 \end{bmatrix}$

13. $BE = NA$ **14.** $DG = \begin{bmatrix} -10 \\ 14 \\ -10 \end{bmatrix}$ **15.** $HD = \begin{bmatrix} 75 & 85 \end{bmatrix}$ **16.** $CG = \begin{bmatrix} -3 \end{bmatrix}$

17. $GC = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$ **18.** $FH = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{bmatrix}$ **19.** $HF = \begin{bmatrix} 3 \end{bmatrix}$ **20.** $AH = NA$

Find the inverse of the following matrices:

1) $A^{-1} = \begin{bmatrix} 3/19 & 2/19 \\ -2/19 & 5/19 \end{bmatrix}$ **2) does not exists** **3)** $A^{-1} = \begin{bmatrix} 19/59 & 15/59 \\ -4/59 & 70/59 \end{bmatrix}$

4) $A^{-1} = \begin{bmatrix} -5/61 & -12/61 \\ 8/61 & 7/61 \end{bmatrix}$ **5)** $A^{-1} = \begin{bmatrix} -32/143 & 81/143 \\ 60/142 & 9/143 \end{bmatrix}$ **6)** $A^{-1} = \begin{bmatrix} -2/9 & -1/3 \\ -5/9 & -4/3 \end{bmatrix}$

Solving a system of equations by $X = A^{-1}b$ **method.**

Answers: 1) $x = 4, \quad y = 3$ **2)** $x = 13, \quad y = -1$ **3) Does not exists.** **4)** $x = -1, \quad y = -3$

Markovian Process

$P = \begin{bmatrix} .7 & .3 \\ .4 & .6 \end{bmatrix}$ $P^2 = \begin{bmatrix} .61 & .39 \\ .52 & .48 \end{bmatrix}$ $P^3 = \begin{bmatrix} .583 & .417 \\ .556 & .444 \end{bmatrix}$

1) *0.61* **2)** *0.583* **3)** *0.39* **4)** *0.417* **5)** *0.48* **6)** *0.444* **7)** 0.*52* **8)** *0.556*

9) $P^2 = \begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} .61 & .39 \\ .52 & .48 \end{bmatrix} = \begin{bmatrix} 2.35 & 1.65 \end{bmatrix} \Rightarrow 2.35$ **10)** $P^3 = \begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} .583 & .417 \\ .556 & .444 \end{bmatrix} = \begin{bmatrix} 2.305 & 1.695 \end{bmatrix} \Rightarrow 2.305$

11) $P^2 = \begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} .61 & .39 \\ .52 & .48 \end{bmatrix} = \begin{bmatrix} 2.35 & 1.65 \end{bmatrix} \Rightarrow 1.65$ **12)** $P^3 = \begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} .583 & .417 \\ .556 & .444 \end{bmatrix} = \begin{bmatrix} 2.305 & 1.695 \end{bmatrix} \Rightarrow 1.695$