Abe Mirza **Exponent and Logarithm**

Algebraic Properties of Logarithms

The following identities hold for any positive a ≠1 and any positive numbers x and y.

Rules	Name		Example	Practice
1		$log_b 1 = 0$	$log_8 1 = 0$	$log_{10} 1 = ?$
2		$log_b b = 1$	$log_{\mathbf{10}} 10 = 1$, $Log_{\mathbf{28}} 28 = 1$	$log_{\mathbf{2}} 2 = ?$
3	**Power**	$log_b a^n = n\,log_b a$	$log_{10} 8^3 = 3 log_{10} 8$	$log_5 2^6 = ?$
4	**Power**	$log_{b^m} a^n = \frac{n}{m} log_b a$	$log_8 128 = log_{2^3} 2^7 = \frac{7}{3} log_2 2 = \frac{7}{3}(1)$	$log_{27} 243 = ?$
5	**Product**	$log_b a + log_b c = log_b ac$	$log_{10} 7 + log_{10} 9 = log_{10} 63$	$log_5 20 + log_5 5 = ?$
6	**Quotient**	$log_b a - log_b c = log_b \frac{a}{c}$	$log_{10} 12 - log_{10} 3 = log_{10} 4$	$log_5 20 - log_5 5 = ?$
7		$b^{log_b a} = a$	$5^{log_5 8} = 8$	$3^{log_3 11} = ?$
8	**Base 10**	When base is 10, we write base as blank.	$log_{10} 81 = log 81$, $log_{10} x^2 = 2 log\, x$	$log_{10} 20 = ?$, $log_{10} yz = ?$
10	**Natural Base (e)**	When base is (e), we write **ln** rather **log**.	$log_e x = \ln x$, $log_e 2 = \ln 2$, $\ln e = 1$	

Steps to solve an exponential Equation:

1. Isolate the exponential part

2. Take **ln** (natural log) from both sides

3. Apply the property of the **ln**

4. Solve for $\boldsymbol{x}$,

Ex 1. Solve for x, $3^{x-4} + 4 = 62$

1. $3^{x-4} = 58$

2. $\ln 3^{x-4} = \ln 58$

3. $(x-4)\ln 3 = \ln 58$

4. $(x-4) = \frac{\ln 58}{\ln 3} = \frac{4.06}{1.099} = 3.694$, $\boldsymbol{x} = 7.694$

Ex 2. Solve for x, $4^{x+2} = 6^{x-4}$

1. $\ln 4^{x+2} = \ln 6^{x-4}$

2. $(x+2)\ln 4 = (x-4)\ln 6$

3 $(x+2)1.386 = (x-4)1.791$, $1.386x + 2.772 = 1.791x - 7.164$,

4. $4.392 = 0.405x$ $\boldsymbol{x} = 10.84$

Ex 3 Solve for x, $7 \log x = 13$

1. Isolate the log part $\log x = 13/7 = 1.857$

2. Use property of log, $\boldsymbol{x} = 10^{1.857} = 71.945$

Ex 4 Solve for x, $7 \ln x = 13$

1. Isolate the log part $\ln x = 13/7 = 1.857$

2. Use property of ln, $\boldsymbol{x} = e^{1.857} = 6.404$

Ex 5. Solve for x, $4\log(x+3) = 11$ **1.** $\log(x+3) = 11/4 = 2.75$ **2.** $(x+3) = 10^{2.75} = 562.34$, $\boldsymbol{x} = 559.34$

Practice Problems: Solve for x

1. $3^x - 5 = 12$ **2.** $2^x + 4 = 15$ **3.** $7^x - 35 = 12$ **4.** $3^{x-4} + 4 = 62$ **5.** $3^x = \left(4^{3x-5}\right)$

6. $3^{2x-4} - 11 = 18$ **7.** $3^{x+1} = 5^{x-1}$ **8.** $3^{x+1} = (1/5)^{x-1}$ **9**. $5\log x = 7$ **10**. $5\ln x = 7$

11. $\log x^2 = 4.67$ **2.** $\ln x = 4.67$ **13.** $3\log\left(x-2\right) = 5.7$ **14**. $3\ln\left(x-2\right) = 5.7$

15. $3^{2x-3} = 51$ **16**. $13^{2x-3} = 51^{3x-2}$ **17**. $1 = \dfrac{2005}{3+4e^{5x+6}}$ **18**. $log_{27}\dfrac{1}{81} = x$ **9**. $\log_2 (x+6) = 3$

20. $8(4^{6-2x}) + 13 = 41$ **21**. $\log_{10}^{(7x-6)} - \log_{10}^{x} = \log_{10}^{(x+2)}$ **22.** $\log_{10}^{4x} - \log_{10}^{(1-x)} = 2$ **23.** $\left(16 - \dfrac{.878}{26}\right)^{9x} = 21$

24. $e^{2x} + 9e^x - 36 = 0$ **25.** $\ln(x-1) + \ln 4 = \ln(2x+4) - \ln 2$ **26**. $e^{x+1} = 28e^{2x-1}$

27. $3(2^{2x+3}) - 4^{2x+3} = 2$ **28.** $\dfrac{6}{e^x} + e^x - 7 = 0$ **29**. $\ln(2x-1) + \ln 4 = \ln(x+4) - \ln 5$

30. Given, $f(t) = 5000 - 100e^{0.15t}$ find the t value that will make $f(t) = 2020$

31. Given, $f(t) = \dfrac{2020}{500 - 8e^{0.15t}}$ find the t value that will make $f(t) = 5$

Applications

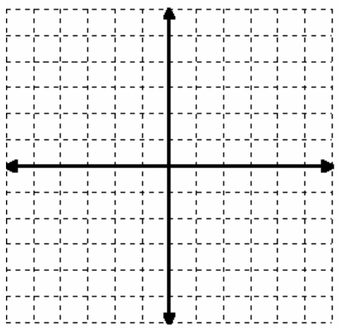
The length in (centimeter) of a typical Pacific habitat t yr old is approximately $f(t) = 32(1 - 0.154e^{-0.17t})$
32. After how long the length will reach 55 centimeters?
33. After how long the length will reach 105 centimeters?

The height of a tree (in feet) is given by $h(t) = \dfrac{200}{1+320e^{-0.27t}}$ where t is the age of a tree in years?

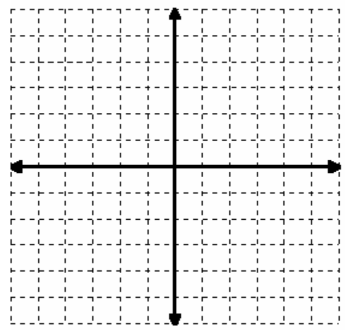
34. What will be the height if the tree is 10 years old?
35. After how long the height will reach 50 feet?
36. After how long the height will reach 80 feet?
Graph each function **without** using **calculator**

Graph each function and find the x and y intercepts by labeling their coordinates on the graph.

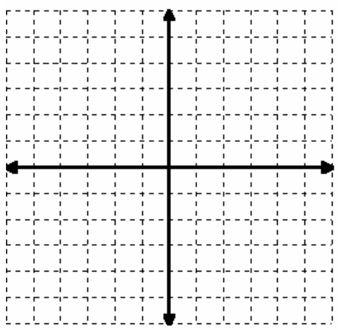
1) $f(x) = 2 - 3^{x}$



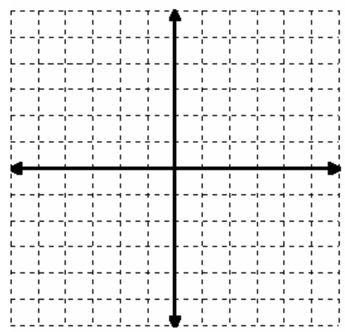
2) $f(x) = 3 + 2^{-x}$



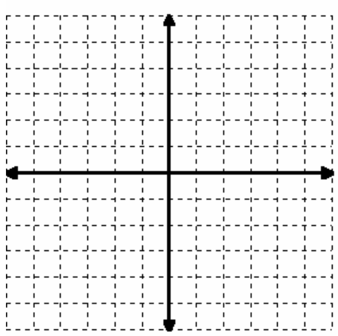
3) $f(x) = -2 + 3^{x}$



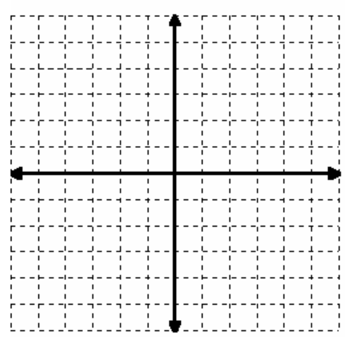
4) $f(x) = 2 - 3^{-x}$



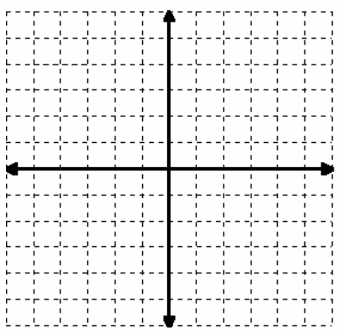
4) $f(x) = \ln(x-2)$



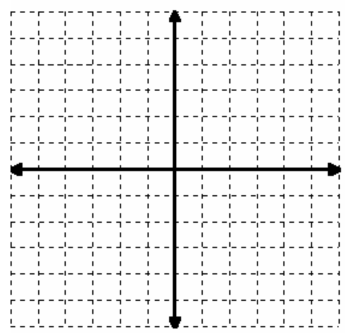
5) $f(x) = \ln(x+2) - 3$



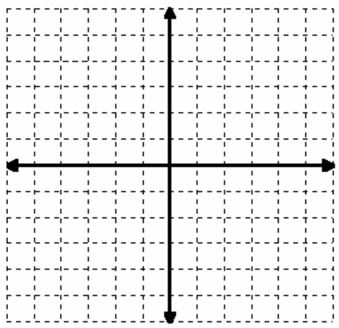
6) $f(x) = -3 - 3^{x-3}$



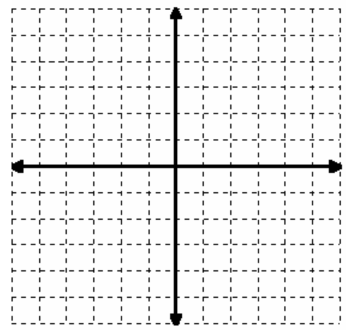
7) $f(x) = 4 - e^{-x-4}$



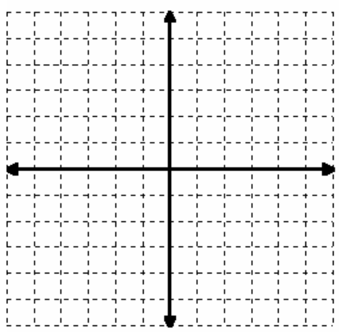
8) $f(x) = 4 + 2^{-x+2}$



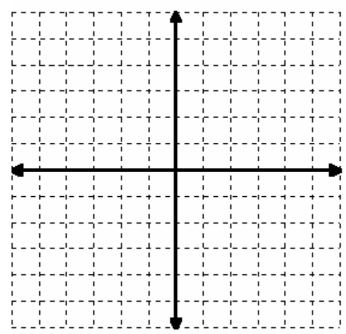
9) $f(x) = -3 - \ln(x-3)$



10) $f(x) = 3 - \log(-x+3)$



11) $f(x) = -3 + \log(x+3)$



Answers

1) $x = 2.58$	**2)** $x = 3.45$	**3)** $x = 1.979$	**4)** $x = 7.69$	**5)** $x = 2.265$
6) $x = 3.532$	**7)** $x = 5.2986$	**8)** $x = 0.188$	**9)** $x = 25.11$	**10)** $x = 4.055$
11) $x = 216.27$	**12)** $x = 106.7$	**13)** $x = .8143$	**14)** $x = 8.68$	**15)** $x = 3.289$
16) $x = 0.0254$	**17)** $x = 0.0431$	**18)** $x = 4/3$	**19)** $x = 2$	**20)** $x = -2.548$
21) $x = 2, 3$	**22)** $x = 25/26$	**23)** $x = 0.122$	**24)** $x = 1.099$	**25)** $x = 2$
26) $x = -1.33$	**27)** $x = -1.5, 1$	**28)** $x = 0, 1.79$	**29)** $x = 24/39$	**30)** $t = 22.63$
31) $t = 16.905$	**32)** $t = 16.905$	**33)** $t = 15.85$	**34)** $h = 8.8$	**35)** $t = 17.29$
36) $t = 19.86$				

Exponential Function

$$F = P e^{rt}$$

$r > 0$ **Exponential Growth** $r < 0$ **Exponential Decay**

To solve for t or r, first use $r\,t = \ln(\frac{F}{P})$ and then solve for t or r,

Ex 1: If the world population grows exponentially at a rate of 2.5% per year and now the population is 6 billion, then

1. What will be world population after **10 years**? $F = P e^{rt} = 6e^{.025(10)} = 6e^{.25} = 6(1.284) = 7.7$ *billion*

2. What will be the world population after **25 years**? $F = P e^{rt} = 6e^{.025(25)} = 6e^{.625} = 6(1.868) = 11.21$ *billion*

3. After how long the population will be **7 billion**? $.025t = \ln(7/6) = 0.15415$, $.025t = 0.15415$ $t = 0.15415/.025 = 6.17$

4. After how long the population will be **doubled**? $.025t = \ln(2) = 0.6931$ $.025t = 0.6931$ $t = 0.6931/.025 = 27.73$ *years*

5. After how long the population will be **tripled**? $0.025t = \ln(3) = 1.0986$ $.025t = 1.0986$ $t = 1.0986/.025 = 43.94\,yrs$

Ex 2: If the world population was 5 billion at year 1990 and it reached 6 billion at year 2002. Assuming the world population grows exponentially then

1. What is the growth rate of the world population? $F = P e^{rt}$ $6 = 5\,e^{r\,12}$ $6/5 = e^{r\,12}$
$\ln(6/5) = 12r$ $.18232 = 12r$ $r = .01519 = 1.52\%$
2. What will be population in year 2010? $t = 2010 - 1990 = 20$ $F = P e^{rt} = 5e^{0.0152(20)} = 5e^{0.304} = 5(1.355) = 6.78$

3. What will be the population in year 2015? $t = 2015 - 1990 = 25$ $F = P e^{rt} = 5e^{.0152(25)} = 5e^{0.38} = 5(1.462) = 7.27$

4. After how long the pop. will be 7 billion? $.0125t = \ln(7/5) = 0.3365$ $.0152t = 0.3365$ $t = 0.3365/.0152 = 22.14$

5. After how long the population will be **doubled**? $.0152t = \ln(2) = 0.6931$ $.025t = 0.6931$ $t = 0.6931/.0152 = 45.6$

6. After how long the population will be **tripled**? $.0152t = \ln(3) = 1.0986$ $.0152t = 1.0986$ $t = 1.0986/.0152 = 72.28$

Practice Problems

A: If the world population grows exponentially at a rate of 2.% per year and at the present time the population is 6.2 billion, then
1. What will be the world population 10 years from now?

2. What will be the world population 20 years from now?

3. After how long the world population will be 7.5 billion? **yrs**

4. After how long the world population will be doubled? **yrs**

5. After how long the world population will be tripled? **yrs**

B: If the world population was 5.7 billion at year 1995 and it reached 6 billion at year 2003. Assuming the world population grows exponentially then

1. What is the growth rate of the world population? **6.4%**
2. What will be world population in year 2010?
3. What will be the world population in year 2015?
4. After how long the world population will be 7.2 billion?
5. After how long the world population will be doubled? **yrs**
6. After how long the world population will be tripled? **yrs**

C: If the price of a machine was $20,000 in year 2004 and it was dropped to $18,000 in year 2006. Also assuming the price of a machine depreciate exponentially then

1. What is the depreciate rate for price of the machine?

2. What will be the price of the machine in year 2008?

3. What will be the price of the machine in year 2014? .

4. After how long (and in what year) the price of the machine will be $15,000? **yrs**, **Year 2009.5**

5. After how long (and in what year) the price of the machine will be reduced in half? **yrs, Year 2017**
6. After how long (what year) the price of the machine will be reduced to one fourth of its value? **yrs, Yr 2025**

D. If the population of a certain bacteria grows exponentially at a rate of 1.5% per hour and at he present time there are 2000 bacteria, then

1. How many bacteria will be present after 10 hours?
2. How many bacteria will be present after one day?
3. After how long there will be 3500 bacteria?
4. After how long the number of bacteria will be doubled?
5. After how long the number of bacteria will be tripled?

E: At an annual inflation rate of 7.5%, how long will it take the Consumer Price Index (CPI) to

a) Double? **9.14 yrs**
b) Triple? **14.65 yrs**
C) Will be 25% higher? **2.98 yrs**

F. If the population of a certain bacteria grows exponentially at a rate of 5% per hour and at he present time there are 2000 bacteria, then

1. How many bacteria will be present after 10 hours?
2. How many bacteria will be present after one day?
3. After how long there will be 3500 bacteria?
4. After how long the number of bacteria will be doubled?
5. After how long the number of bacteria will be tripled?