Show all work on exam. Closed book and notes. Each question is worth 20 points for a total of 100 points possible. Do well!

1. Put the correct short answer in the blank.
   a. Does \( f(x) = x^2 - 3x - 10 \) have a maximum or a minimum? \( \text{minimum} \uparrow \)
   b. List all possible rational zeros of \( g(x) = 4x^4 + 7x^2 - 2 \)
      \[ \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2 \]
      \[ \pm 1, \pm \frac{1}{2}, \pm 4 \]
   c. Does the \( f(x) = \frac{x^2 - 4}{x^2} \) have a vertical asymptote? \( \text{No} \) (it has a "hole")
      \[ f(x) = \frac{(x-2)(x+2)}{x-2} \]
   d. How would you transform (shift, reflection, stretch/compress) the graph of \( f(x) = \frac{1}{x} \) to graph
      \( h(x) = \frac{2}{x-1} - 3 \)? Shift 1 unit to the right, 3 units down, stretch by factor 2
   e. What value does \( f(x) = \frac{1}{x-3} \) approach as \( x \to 3^- \)? \( -\infty \) As \( x \to \infty \)? 0
      \[ x = 2.9, 2.99, 2.999 \]
      \[ \frac{1}{x-3} = \frac{1}{2.999 - 3} = \frac{1}{-0.001} = -1000 \]
   f. Find the polynomial satisfying: 1 and 2i are roots, \( f(0) = -4 \) and degree = 3
      \[ f(x) = x^3 - x^2 + 4x - 4 \]
      \[ f(x) = a_n (x-1)(x-2i)(x+2i) = a_n (x-1)(x^2 + 4) \]
      \[ = a_n (x^3 - x^2 + 4x - 4) \]
      \[ -4 = f(0) = a_n (0^3 - 0^2 + 4(0) - 4) \implies -4 = -4a_n \]

2. Solve \( x^2 \geq 2x + 2 \). The solution in interval notation is: \((-\infty, 1-\sqrt{3}) \cup [1+\sqrt{3}, \infty)\)
   \[ a_n = 1 \]
   \[ x^2 - 2x - 2 \geq 0 \]
   \[ x^2 - 2x - 2 = 0 \] Solve to get boundary

   \[ x = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3} \]
   \[ 1 - \sqrt{3} \] True
   \[ 1 + \sqrt{3} \] False
   \[ 0^2 \geq 2(0) + 2 \quad \text{?} \quad 0 \neq 2 \]
3. The graph of \( f(x) = -x^3 - 5x^2 + 2x + 10 \) is shown in the figure.

a. Based on the graph of \( f \) and the Rational Zero Theorem, find the root of the equation
\[-x^3 - 5x^2 + 2x + 10 = 0 \]
that is an integer. -5 \( x = -5 \) is a factor

b. Use long division to find the other roots.

\[
\begin{align*}
-x^2 + 2 \\
\overline{x+5) -x^3-5x^2+2x+10} \\
-x^3-5x^2 \\
\overline{0+2x+10} \\
2x + 10 \\
\overline{0}
\end{align*}
\]

\[ f(x) = (x+5)(-x^2+2) = -(x+5)(x^2-2) = 0 \]
\[ x+5=0 \quad \text{or} \quad x^2-2=0 \]
\[ x=-5 \quad \text{or} \quad x^2=2 \]
\[ x = \pm \sqrt{2} \]

\{ -5, -\sqrt{2}, \sqrt{2} \}
4. Graph \( f(x) = x^3 + 12x^2 + 21x + 10 \). Clearly show all intercepts and end behavior in your graph.

Possible rational roots: \( \pm 1, \pm 2, \pm 5, \pm 10 \)

Intercepts:
\[
\begin{align*}
\text{f}(1) &= 1^3 + 12(1)^2 + 21(1) + 10 = 1 + 12 + 21 + 10 \neq 0 \\
\text{f}(-1) &= (-1)^3 + 12(-1)^2 + 21(-1) + 10 = -1 + 12 - 21 + 10 = 0
\end{align*}
\]

so \( x = -1 \) is a factor

\[
\frac{x^3 + 12x^2 + 21x + 10}{x+1} = \frac{x^2 + 11x + 10}{x+1}
\]

\[
\frac{x^2 + 11x + 10}{x+1} = \frac{(x+1)(x+10)}{x+1} = x+10, \quad \text{with a hole at } x = -1
\]

\[
f(x) = (x+1)(x^2 + 11x + 10) = (x+1)[(x+1)(x+10)]
\]

\[
= (x+1)^2(x+10)
\]

\( x = -1 \) root of multiplicity 2, touches and turns

\( x = -10 \) root of multiplicity 1, crosses.

Symmetry: \( f(-x) = (-x)^3 + 12(-x)^2 + 21(-x) + 10 \)

\[
= -x^3 + 12x^2 - 21x + 10 \quad \text{no symmetry}
\]

End behavior: as \( x \to \infty \), \( f(x) \propto x^3 \to \infty \) rises right

as \( x \to -\infty \), \( f(x) \propto x^3 \to -\infty \) falls left

\[
y\text{-intercept: } f(0) = 10
\]
5. Graph \( g(x) = \frac{2x-4}{x+3} \). Give the equations of any horizontal or vertical asymptotes.

**Symmetry:** \( g(-x) = \frac{2(-x)-4}{-x+3} = \frac{-2x-4}{-x+3} \equiv g(x) \)

\( g(-x) \neq -g(x) \)

**Asymptotes:** \( x = -3 \) vertical asymptote

**H.A.** \( \lim_{x \to \infty} g(x) = \frac{2x}{x} = 2 \) \( \lim_{x \to -\infty} g(x) = \frac{2x}{x} = 2 \)

**Intercepts:**

**x-intercepts**
\( 0 = \frac{2x-4}{x+3} \) \( \Rightarrow 0 = 2x - 4 \) \( \Rightarrow x = 2 \)

**y-intercepts:**
\( g(0) = \frac{2(0)-4}{0+3} = -\frac{4}{3} \)

\( g(10) = \frac{2(10)-4}{10+3} = \frac{16}{13} \)

\( g(-4) = \frac{2(-4)-4}{-4+3} = -\frac{12}{-1} = 12 \)

**y = 2**